The Dirac composite fermions in fractional quantum Hall effect

Dam Thanh Son (University of Chicago)
Nambu Memorial Symposium
March 12, 2016

A story of a symmetry lost and recovered

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Plan

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- Fractional Quantum Hall Effect (FQHE)

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- Composite fermions

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- The old puzzle of particle-hole symmetry

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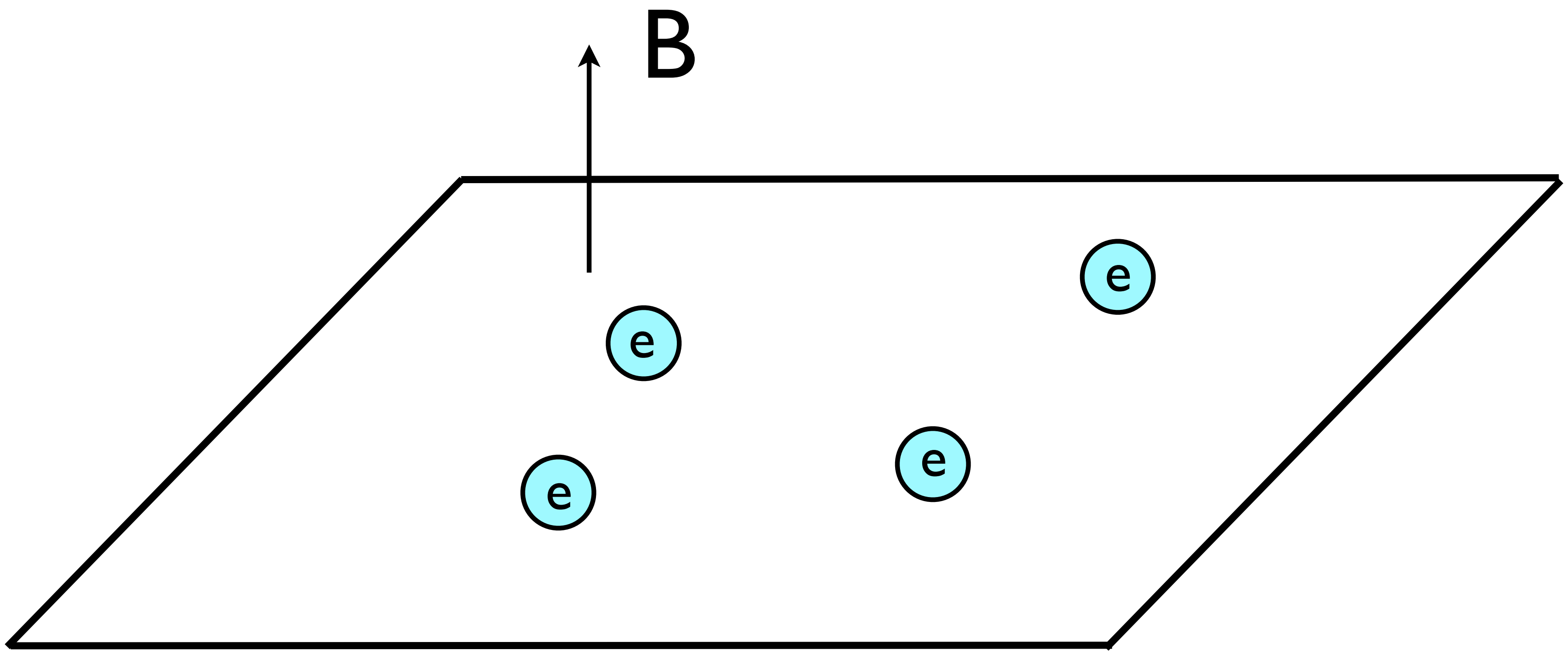
- Fractional Quantum Hall Effect (FQHE)
- Composite fermions
- The old puzzle of particle-hole symmetry
- Dirac composite fermions

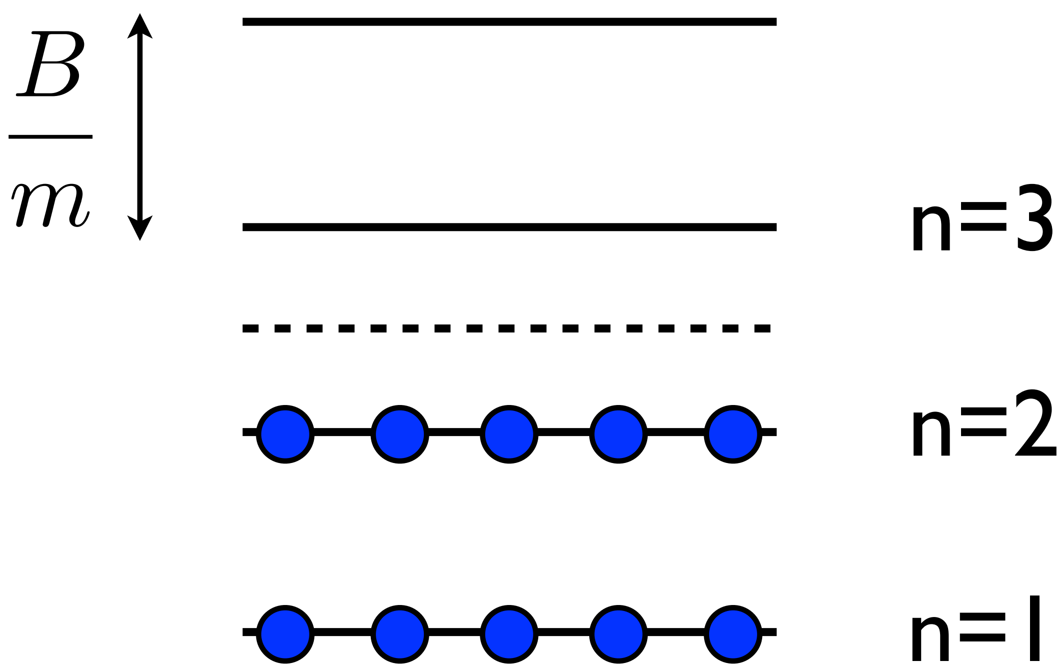
The TOE of QHE



$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

Integer quantum Hall state

- electrons filling n Landau levels

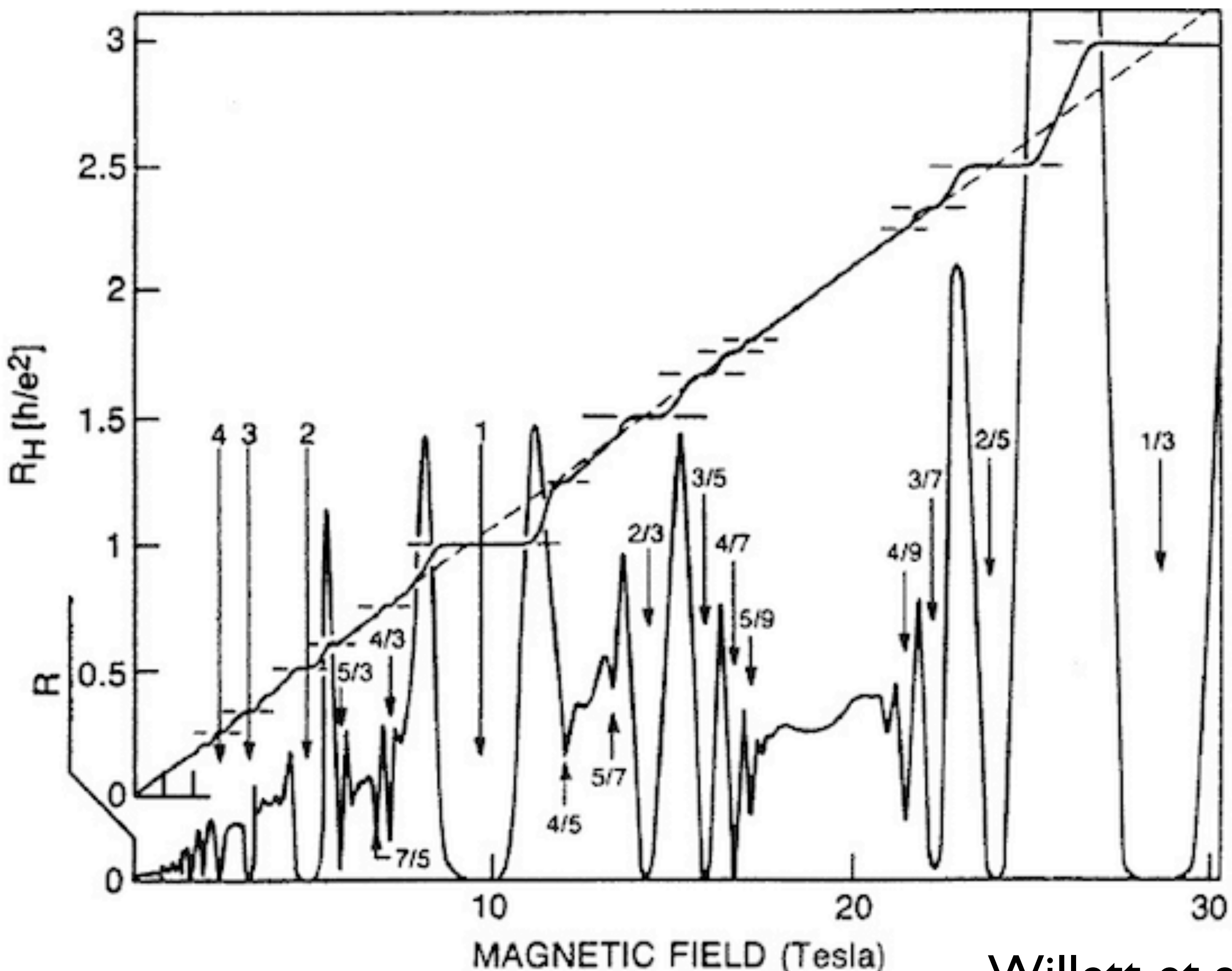


When $\rho = n\frac{B}{2\pi}$

energy gap: $\frac{B}{m}$

1982: Fractional QH effect

(Tsui, Stormer)



Willett et al 1987

$$\sigma_{xy} = \frac{1}{\rho_{xy}} = \frac{p}{q}\frac{e^2}{h}$$

$$j_x = \sigma_{xx}E_x + \sigma_{xy}E_y$$

odd integer (most of the time)

Fractional QHE

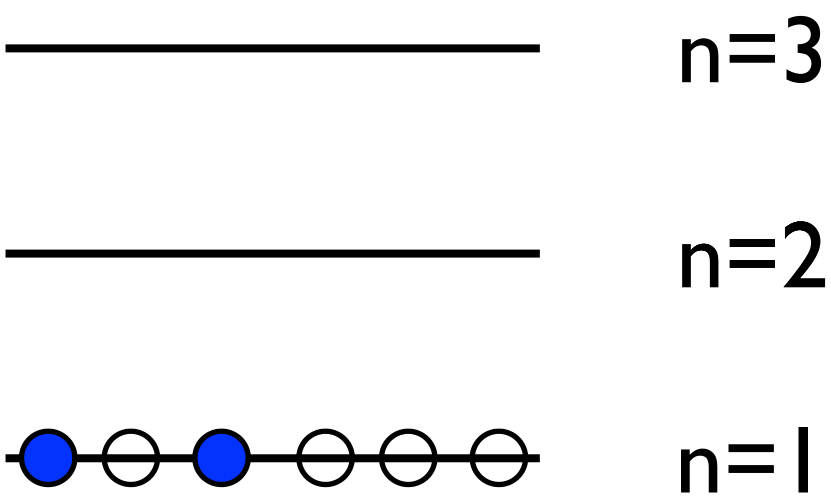
n=3

n=2

n=1

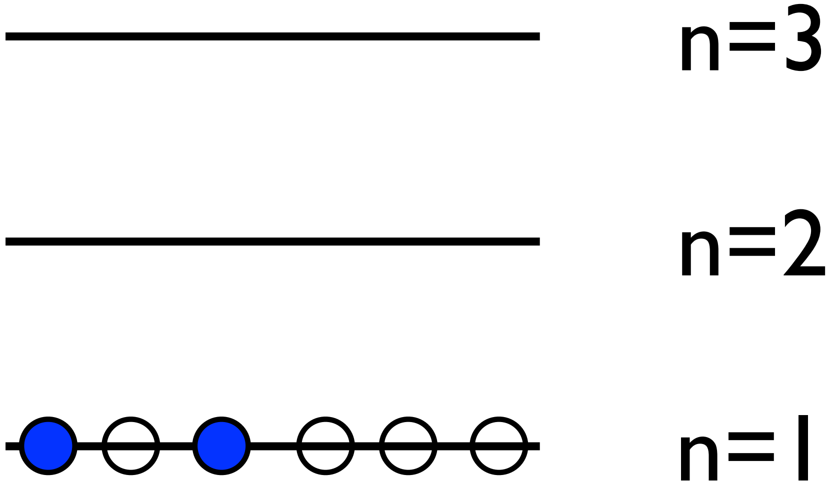
Filling fraction $\nu = \dfrac{\rho}{B/2\pi}$

Fractional QHE



Filling fraction $\nu = \dfrac{\rho}{B/2\pi}$

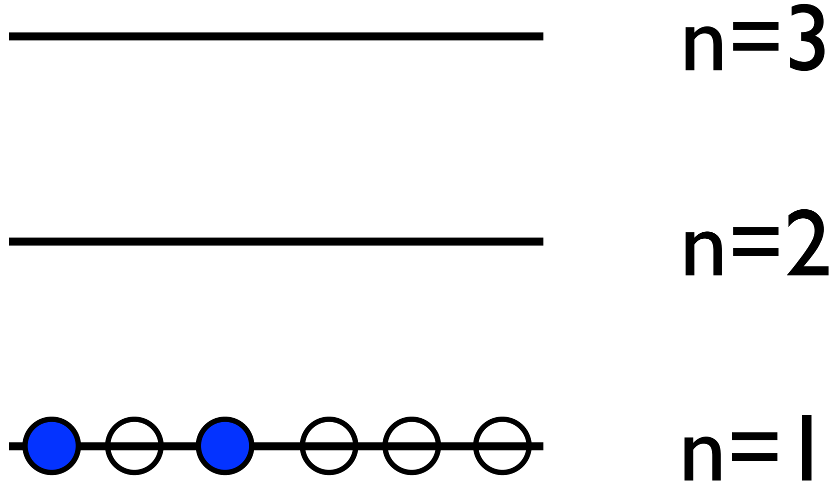
Fractional QHE



Filling fraction $\nu = \dfrac{\rho}{B/2\pi}$

$$\nu = \frac{1}{3}$$

Fractional QHE



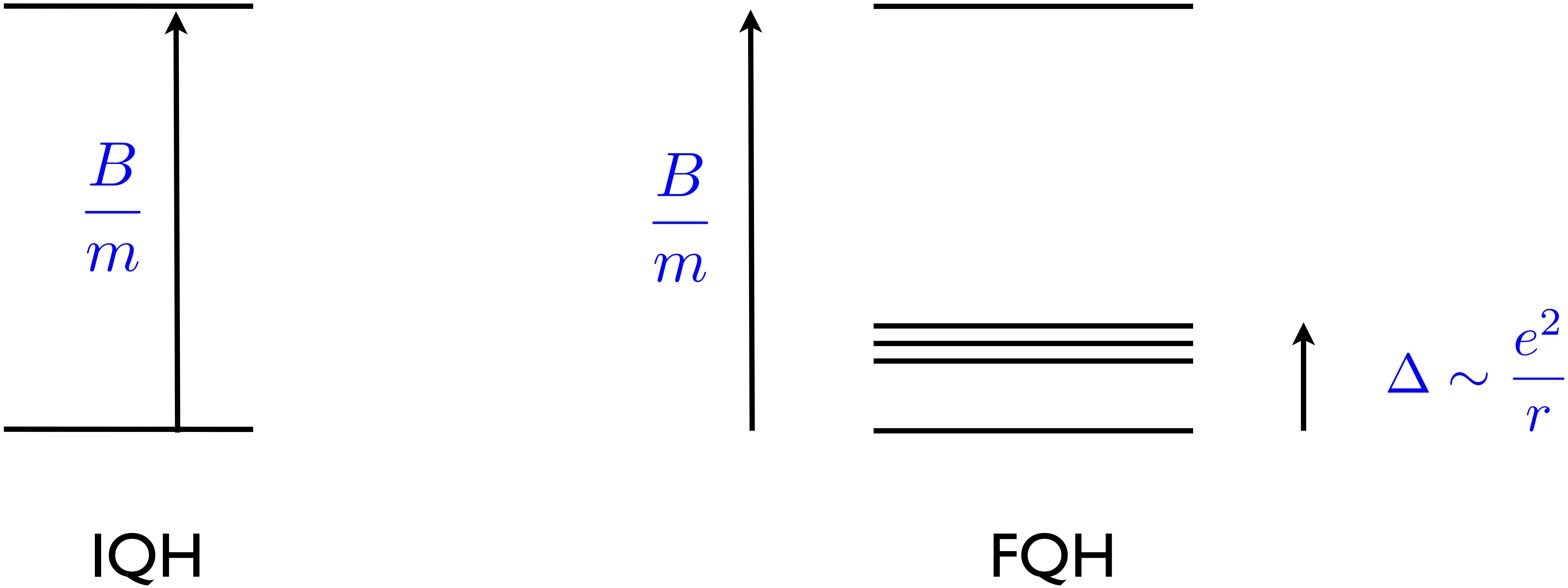
Filling fraction $\nu = \dfrac{\rho}{B/2\pi}$

$$\nu = \frac{1}{3}$$

Large ground-state degeneracy without interactions

Experiments: energy gap for certain rational filling fractions

Energy scales



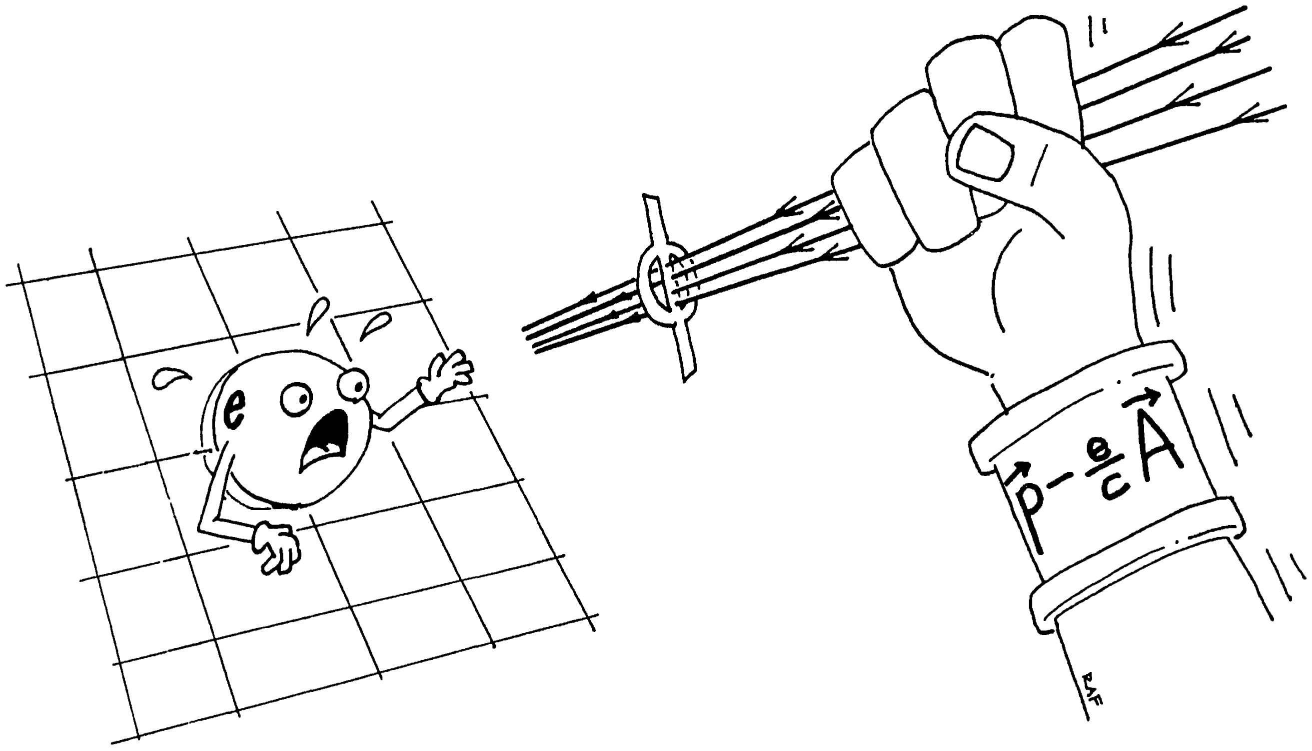
Lowest Landau level limit: B/m >> Δ
Perturbation theory useless

The Standard Model of FQHE

$$H = P_{\mathrm{LLL}} \sum_{a,b} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

Projection to
lowest Landau level

Flux attachment

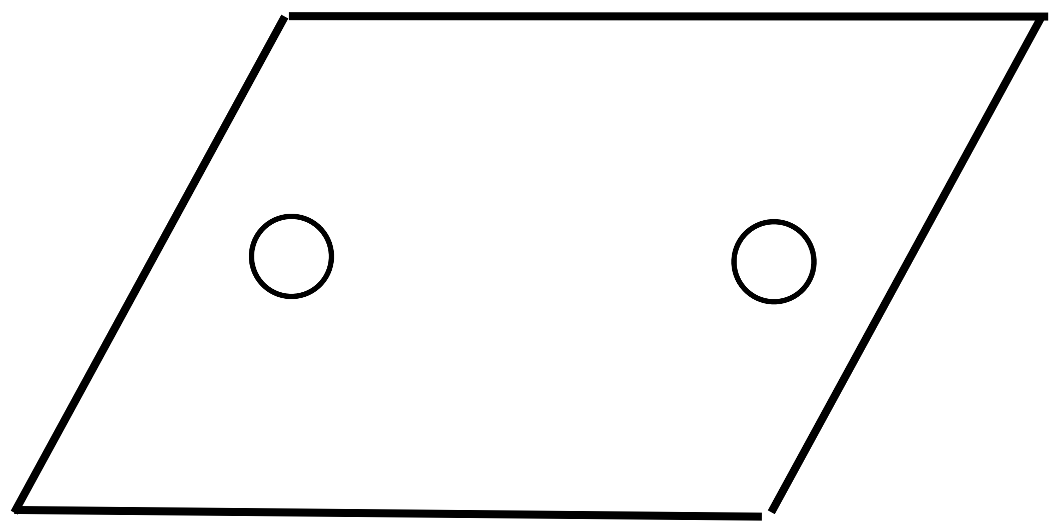


D. Arovas

Flux attachment

(Wilczek 1982, Jain 1989)

- Flux attachment: statistics does not change by attaching an even number of flux quanta



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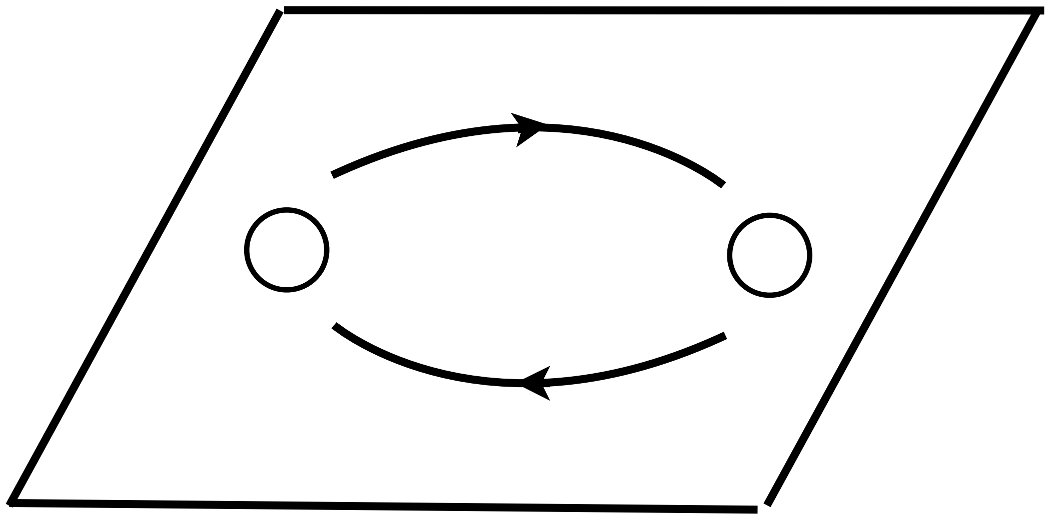

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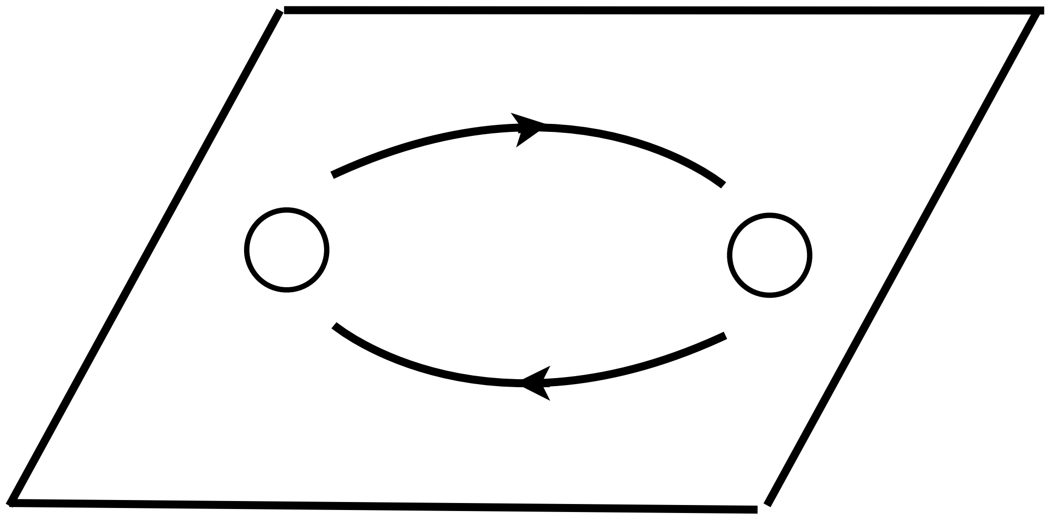

(-1)

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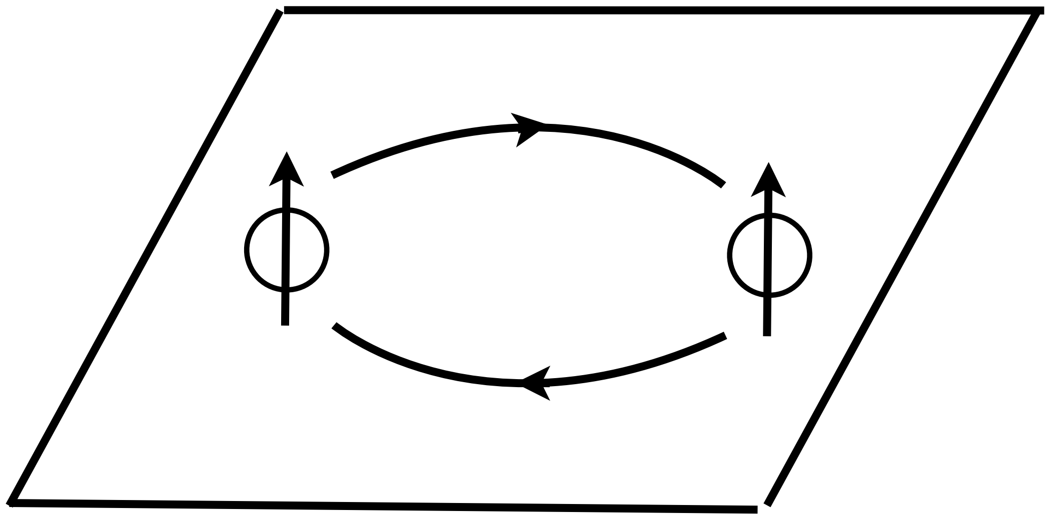

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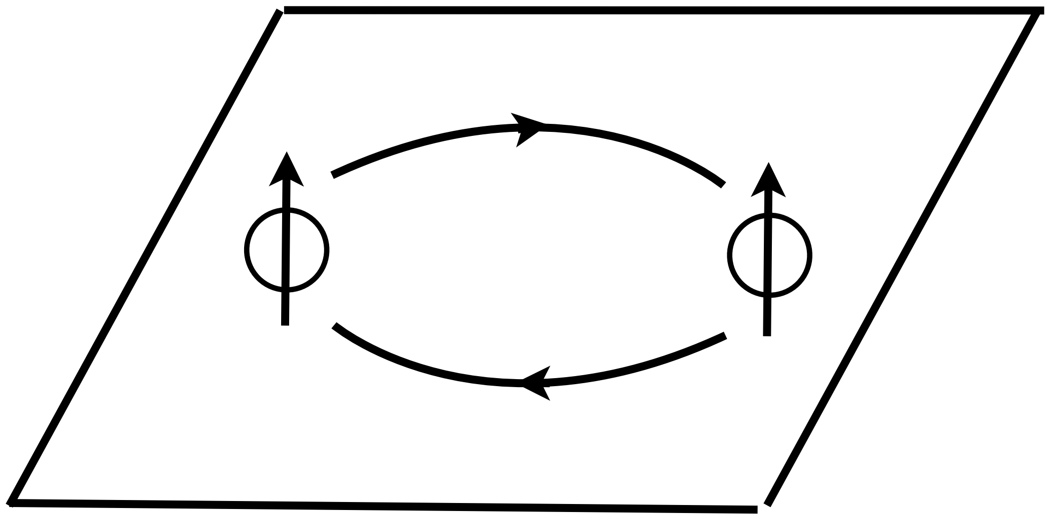

$$(-1)\exp(i\pi) = (+1)$$

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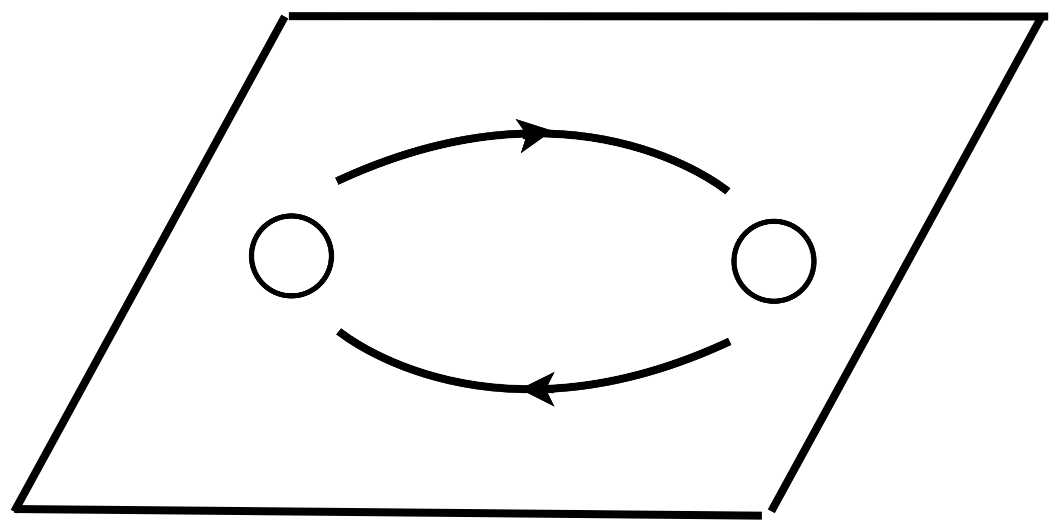

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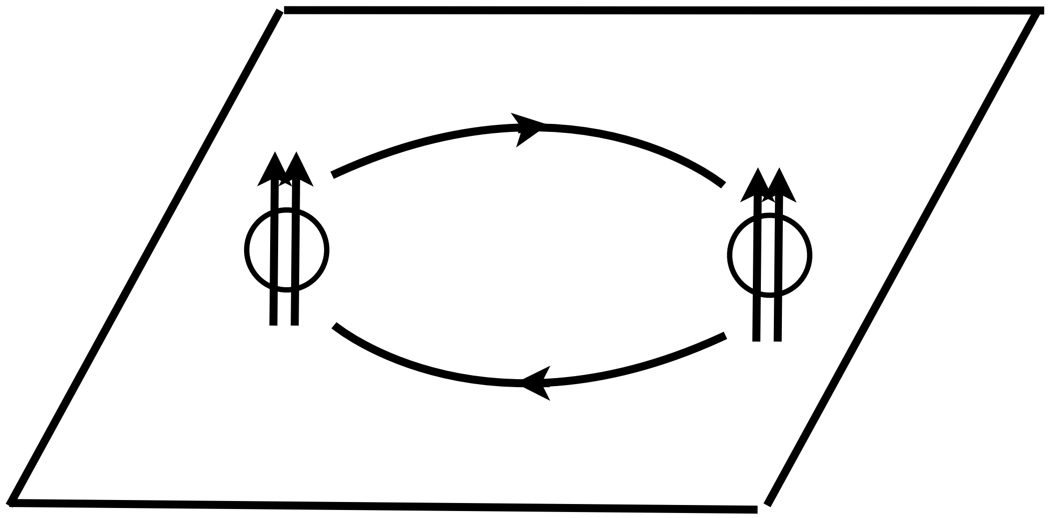

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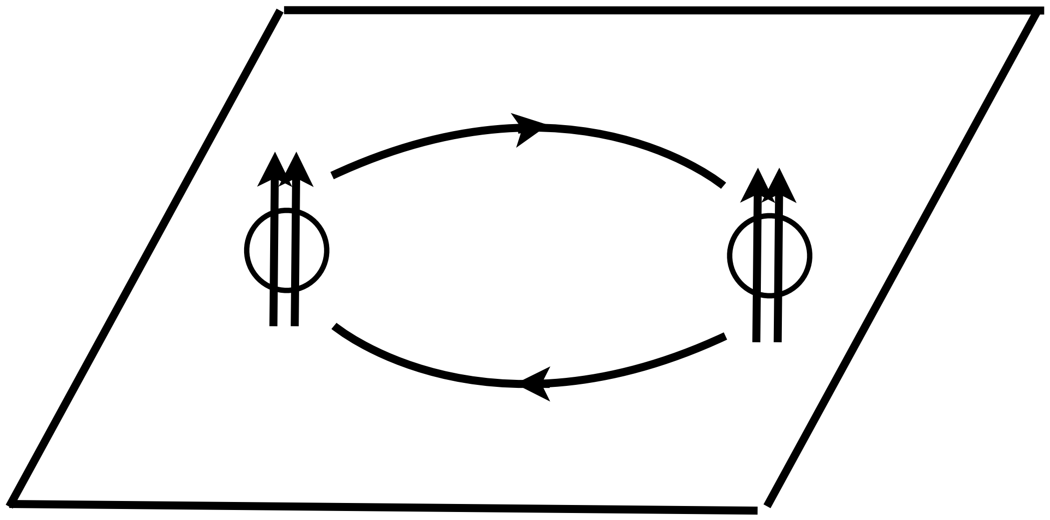

$$(-1)\exp(2i\pi) = (-1)$$

Flux attachment

(Wilczek 1982, Jain 1989)

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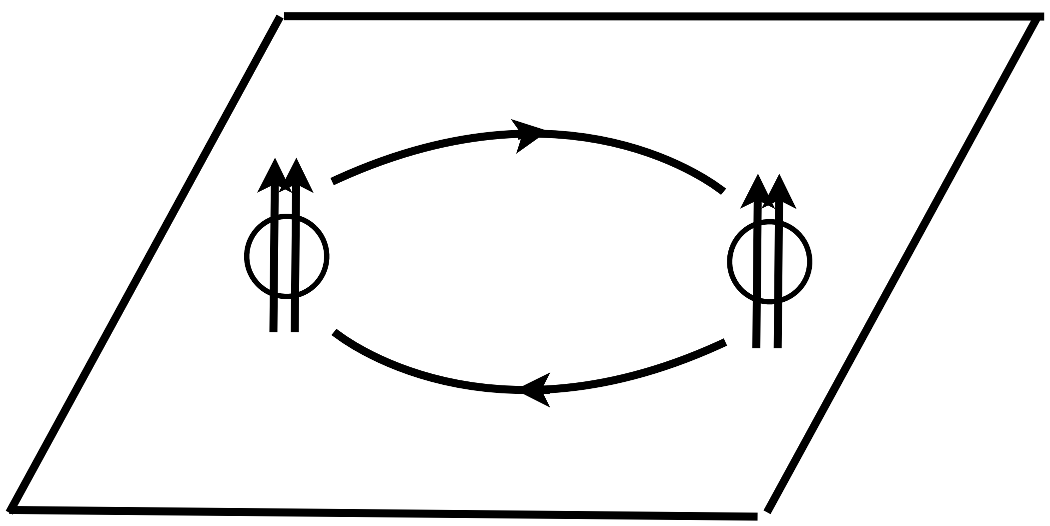

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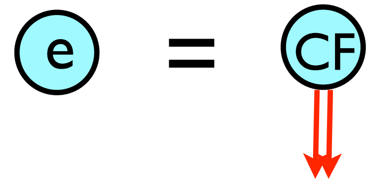
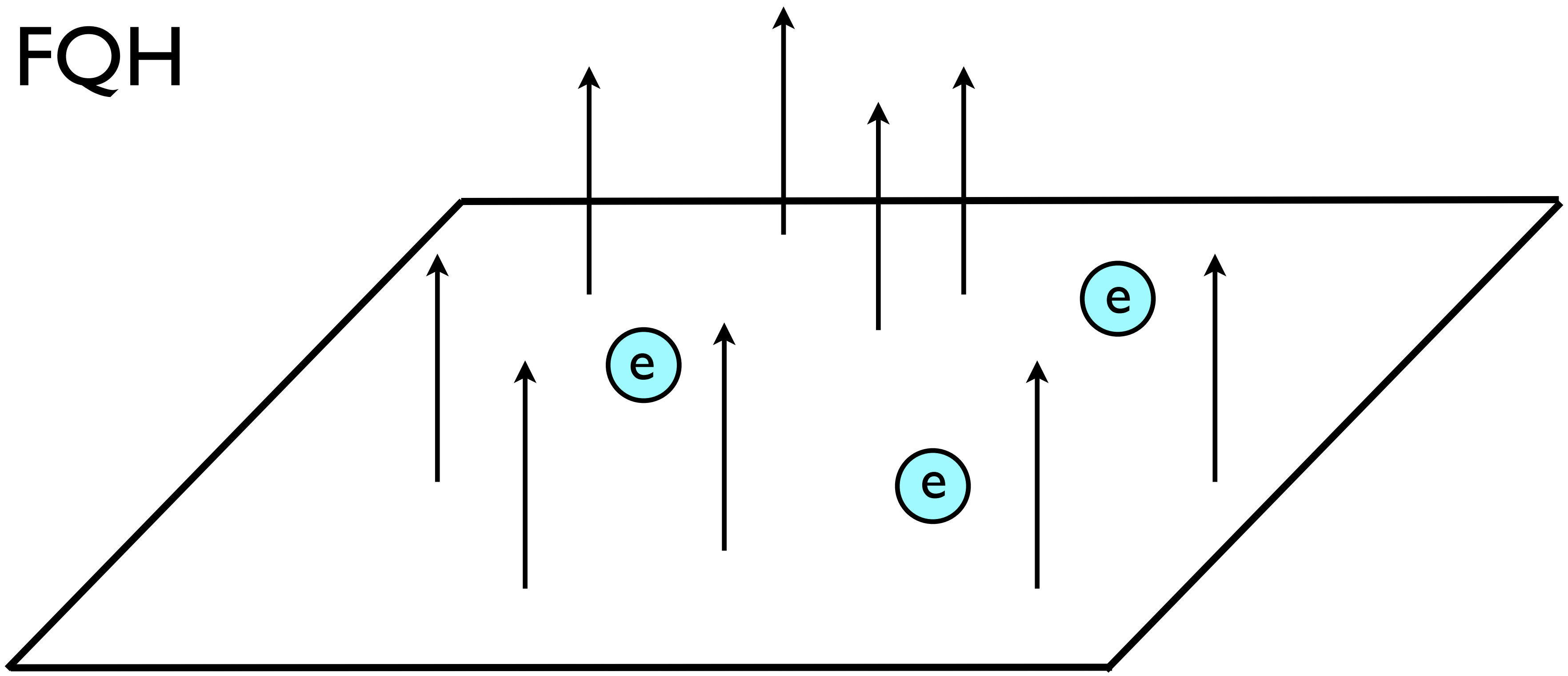
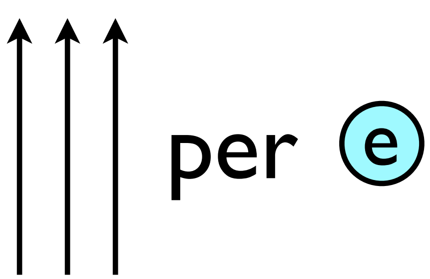


Composite fermion

$\nu = 1/3$ FQH





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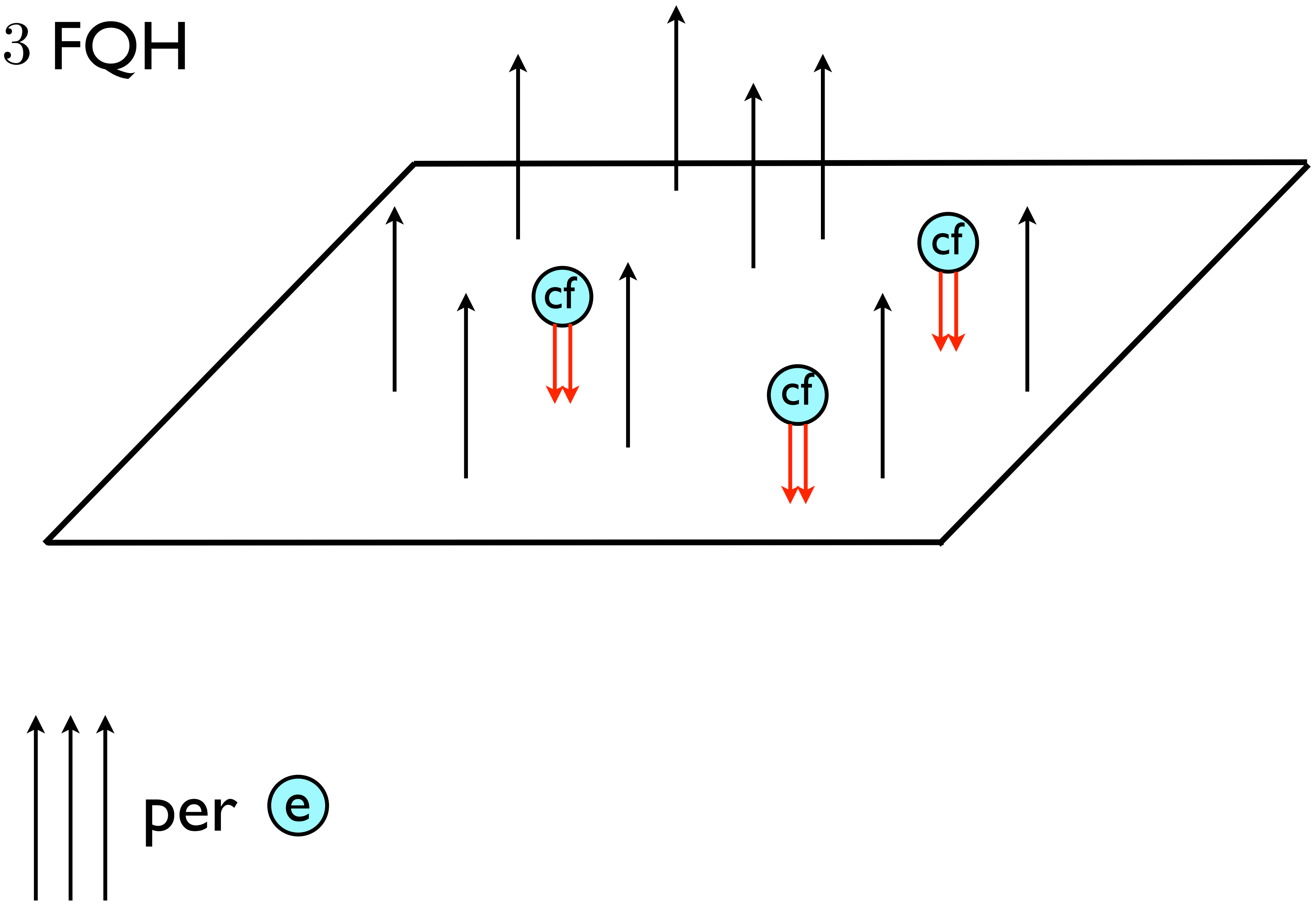

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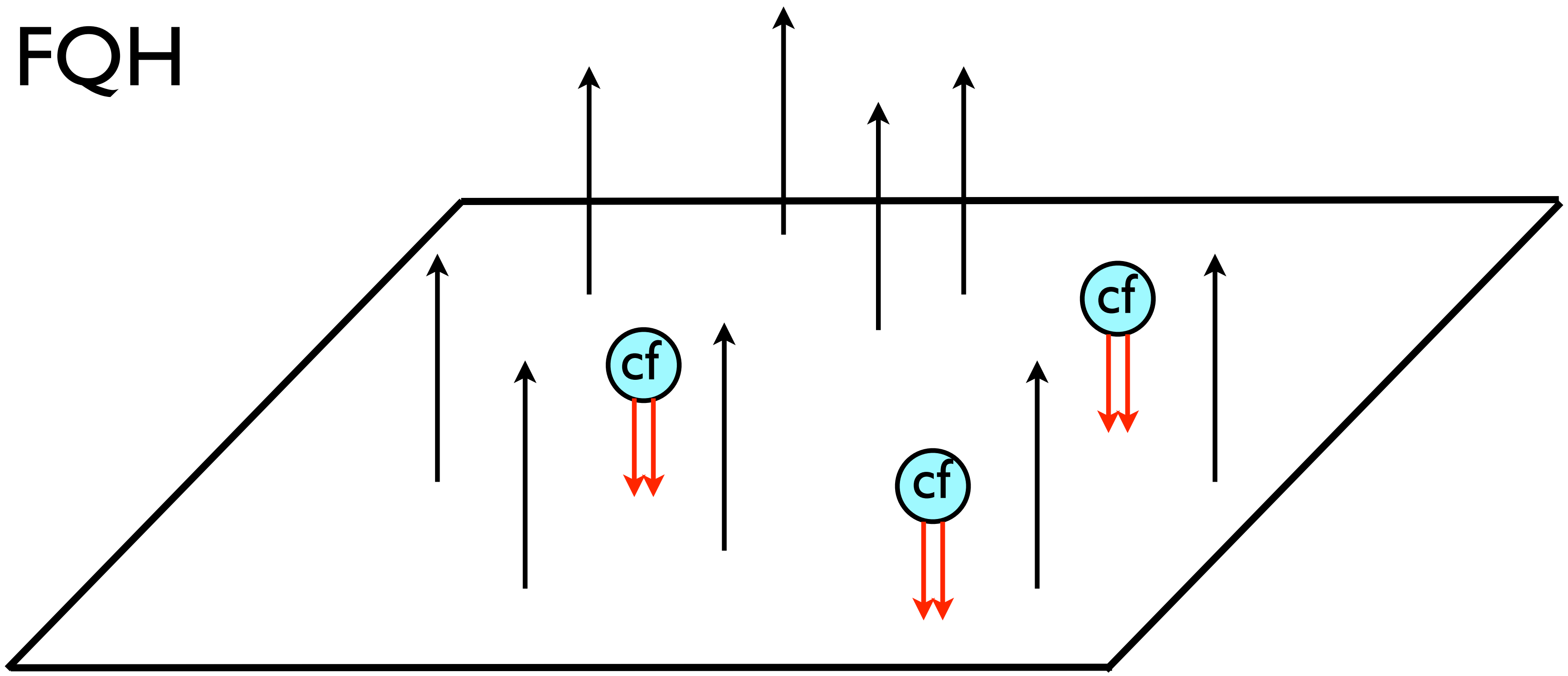

Composite fermion

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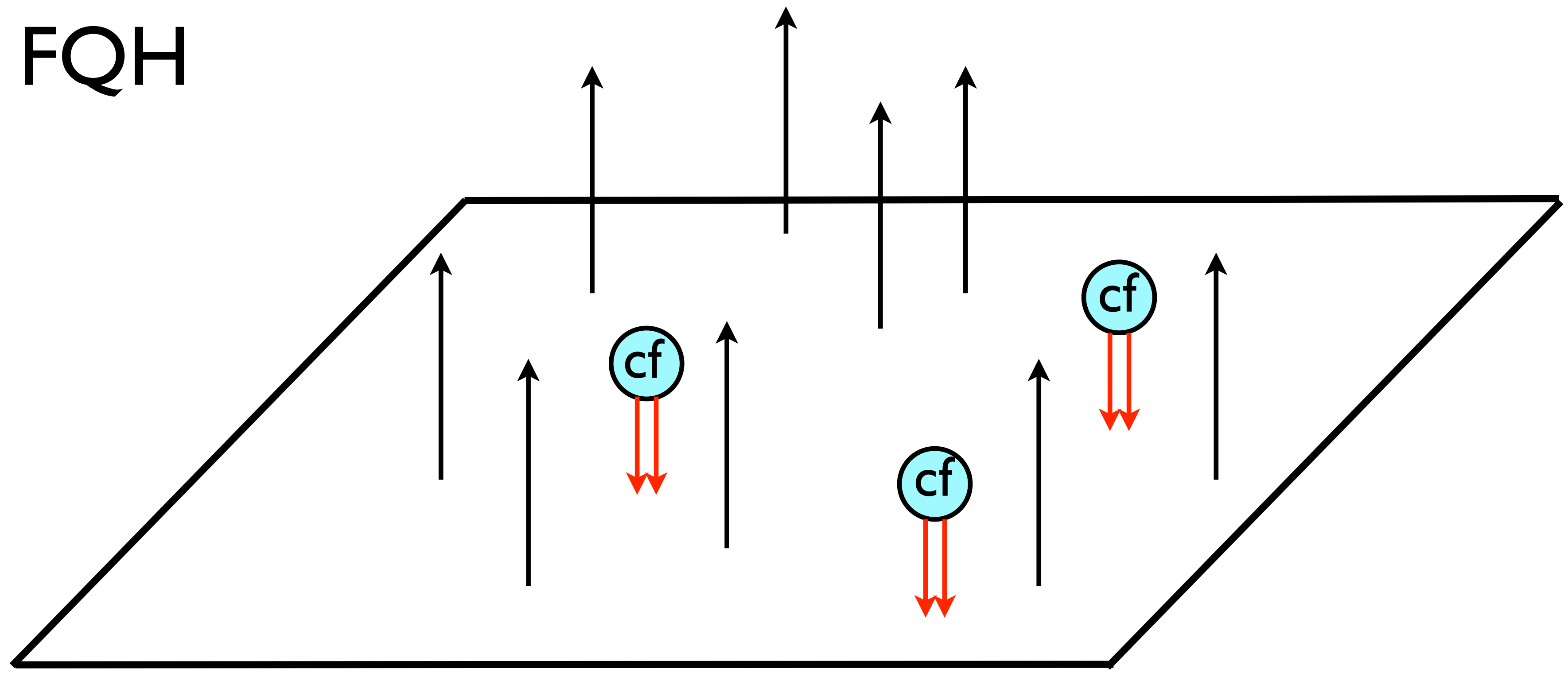


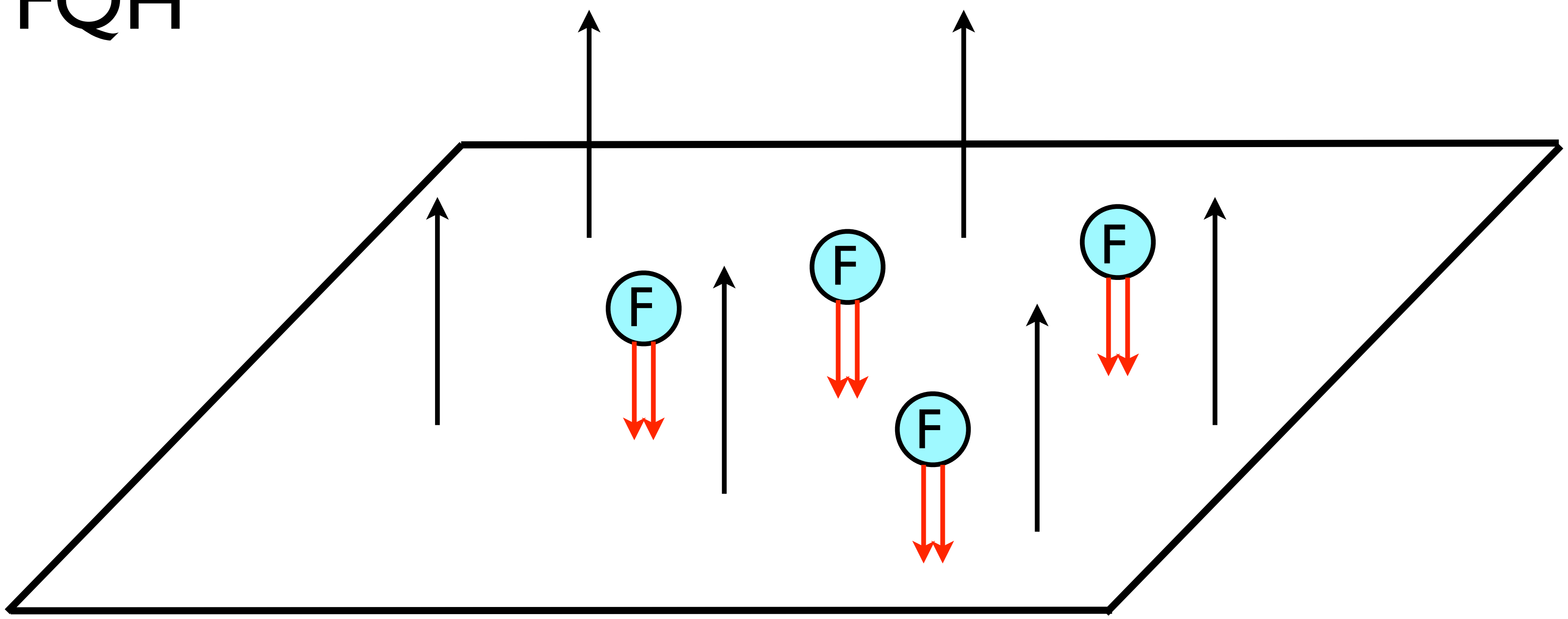
per e

average per cf

IQHE of CFs with $\nu = 1$

Composite fermion

$\nu = 2/3$ FQH



FQHE for
original fermions

=

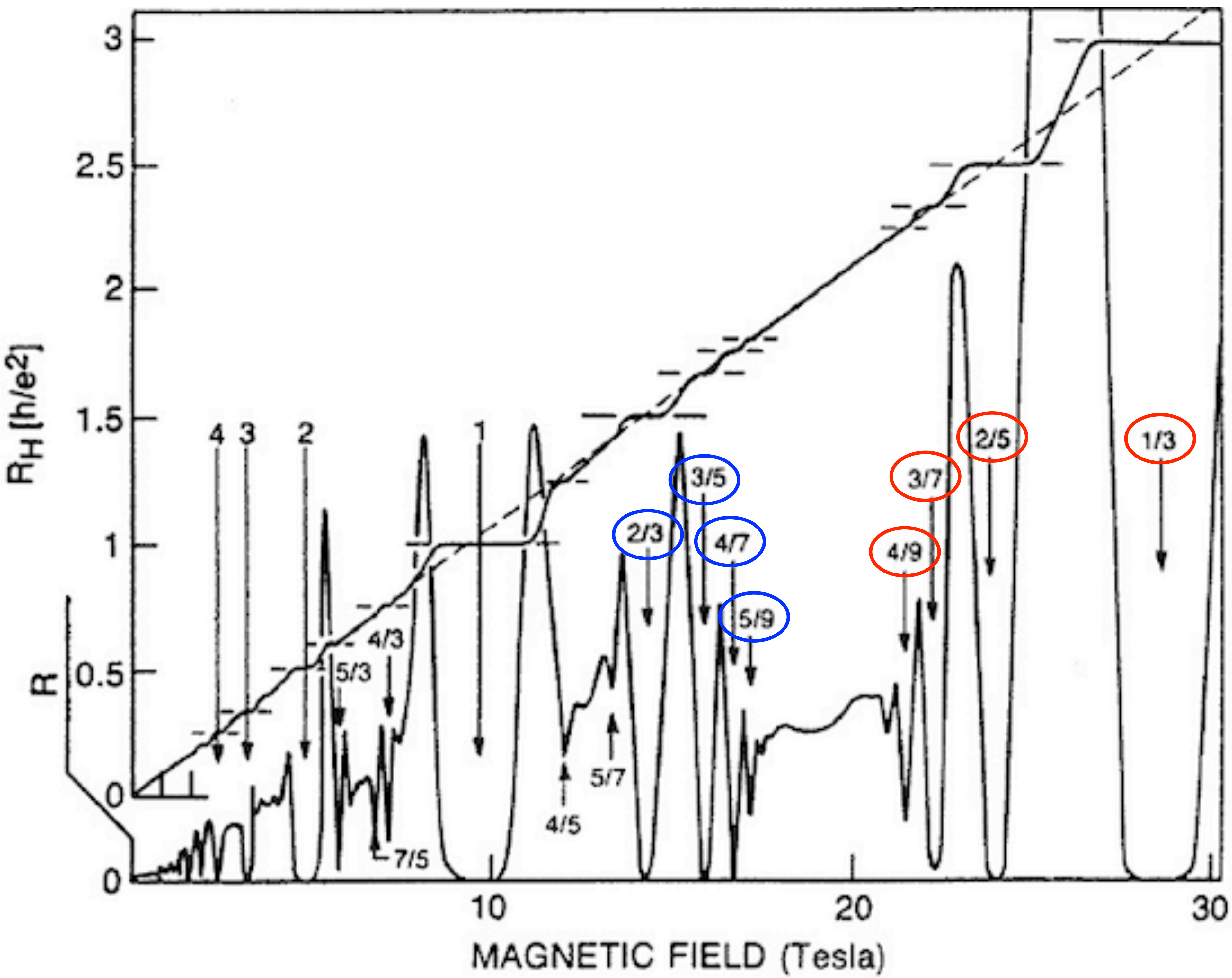
IQHE for
composite fermions (n=2)

Jain's sequence of plateaux

- Using the composite fermion most observed fractions can be explained

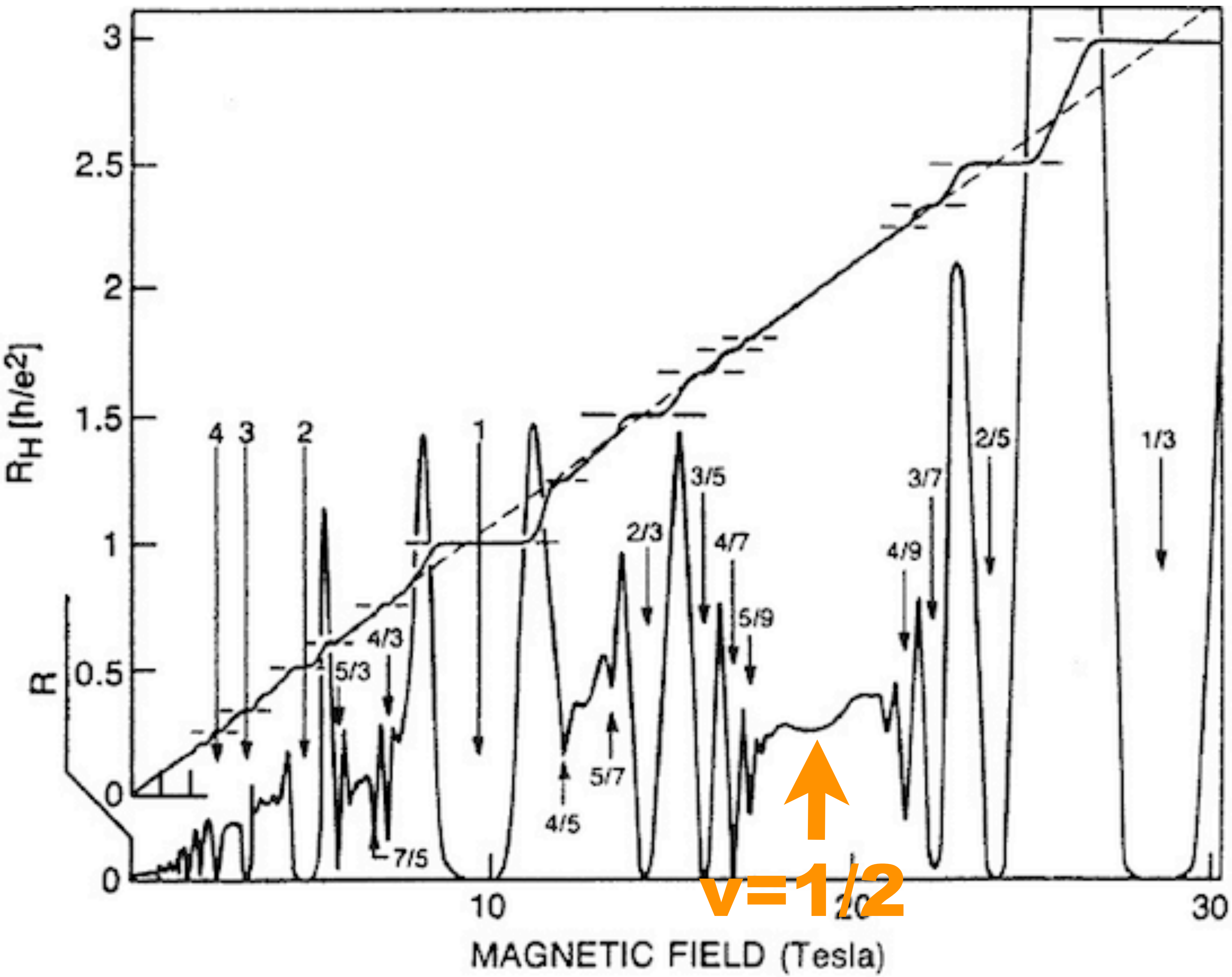
Electrons	Composite fermions
$\nu = \dfrac{n}{2n+1}$	$\nu_{\mathrm{CF}} = n$
$\nu = \dfrac{n+1}{2n+1}$	$\nu_{\mathrm{CF}} = n+1$

Jain's sequences of plateaux



$$\nu = \frac{n+1}{2n+1} \qquad \nu = \frac{n}{2n+1}$$

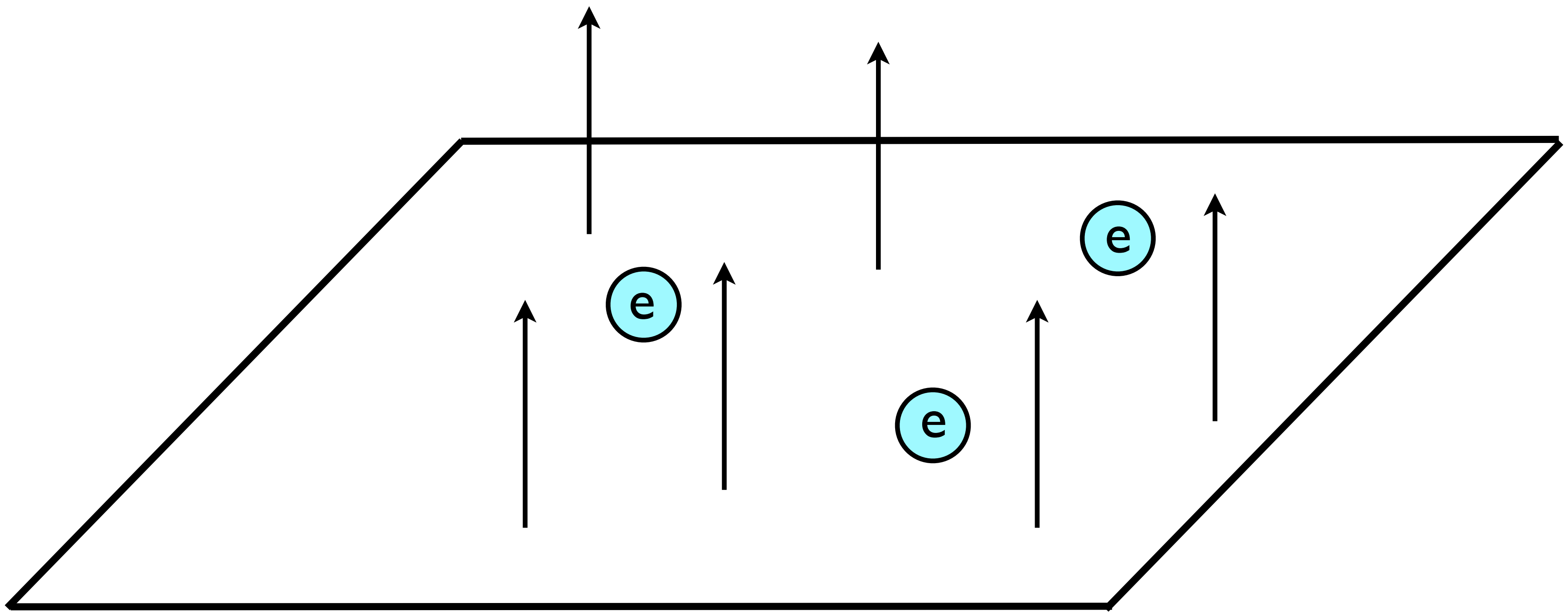
ν=1/2

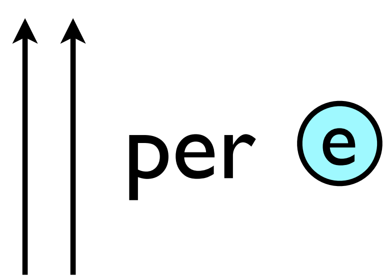


3
2.5
2
1.5
1
0.5
0
0
R_H [h/e^2]
R
4
3
2
1
2/3
3/5
4/7
5/9
4/9
3/7
2/5
1/3
5/3
4/3
7/5
4/5
5/7
ν=1/2
10
20
30
MAGNETIC FIELD (Tesla)

ν=1/2 state

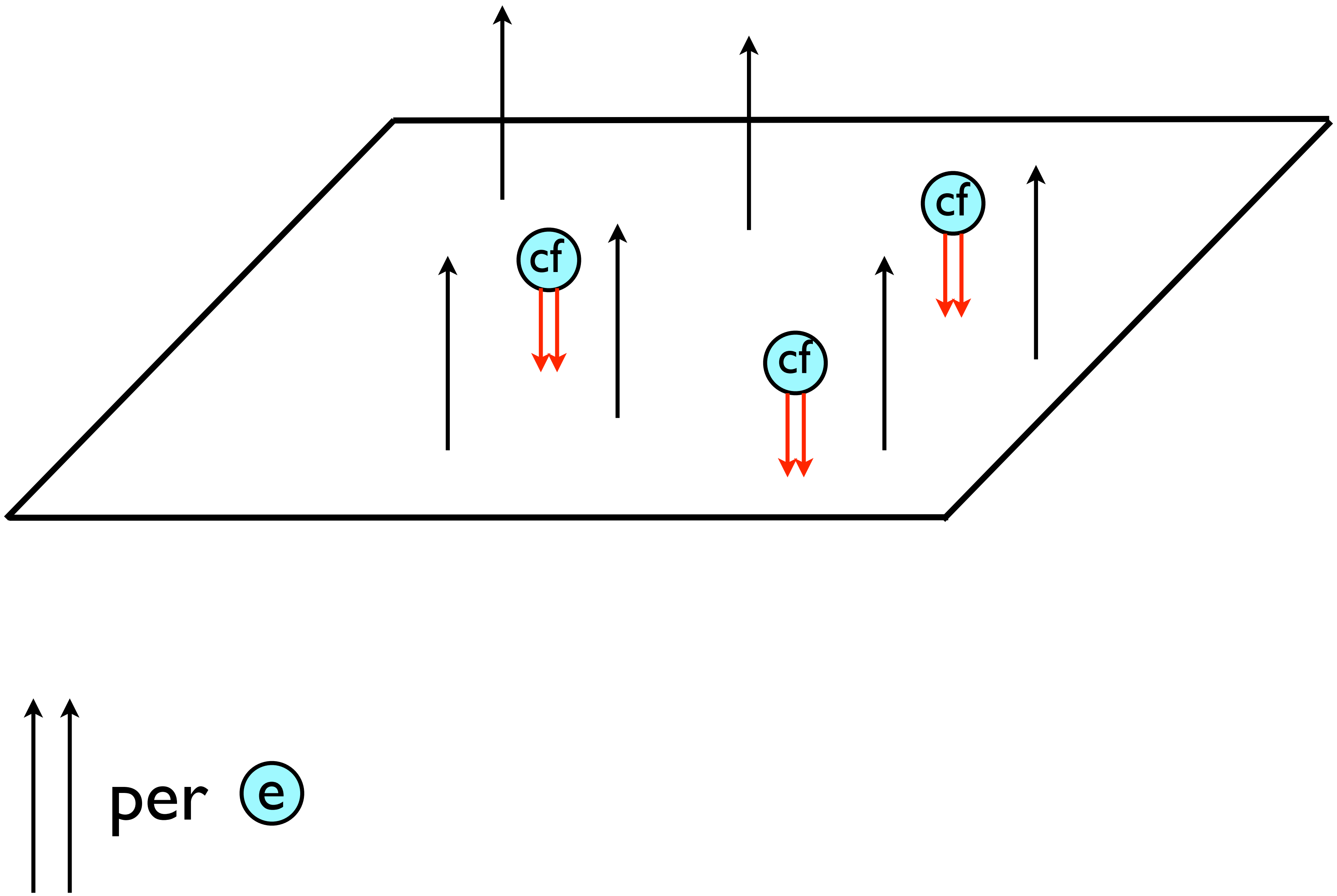
Halperin Lee Read





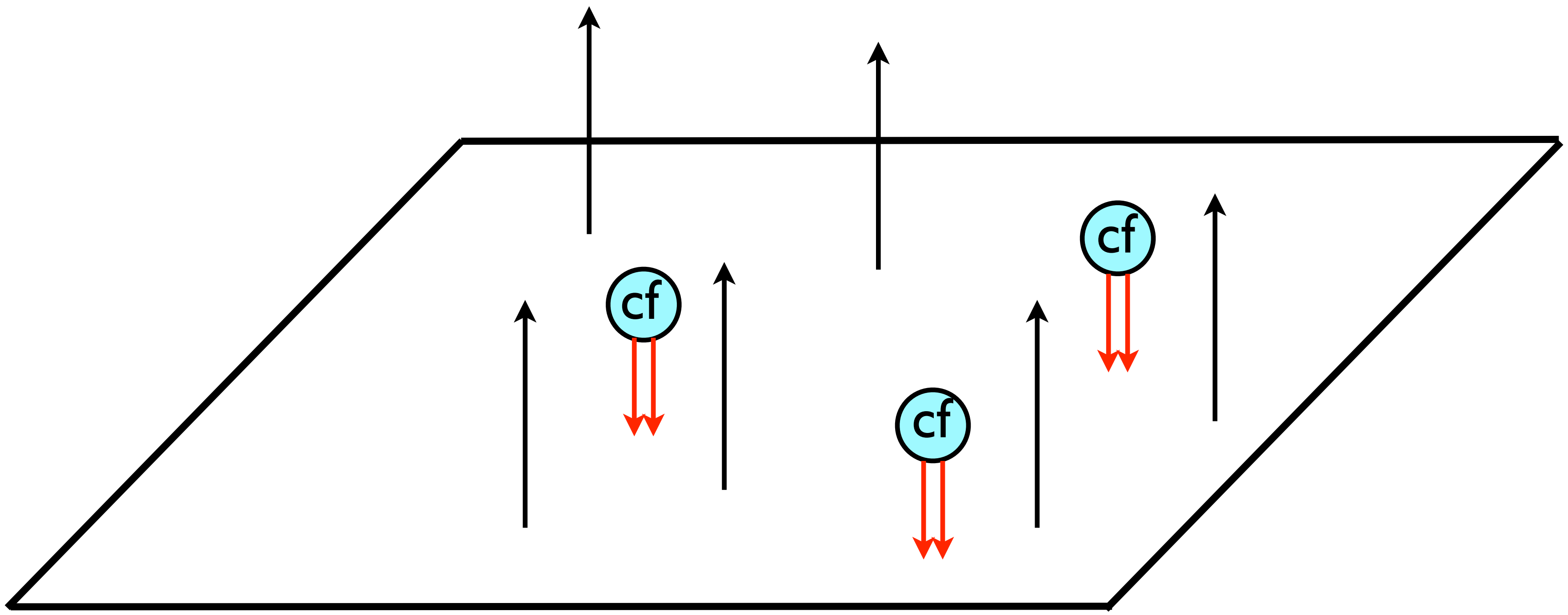
ν=1/2 state

Halperin Lee Read



ν=1/2 state

Halperin Lee Read

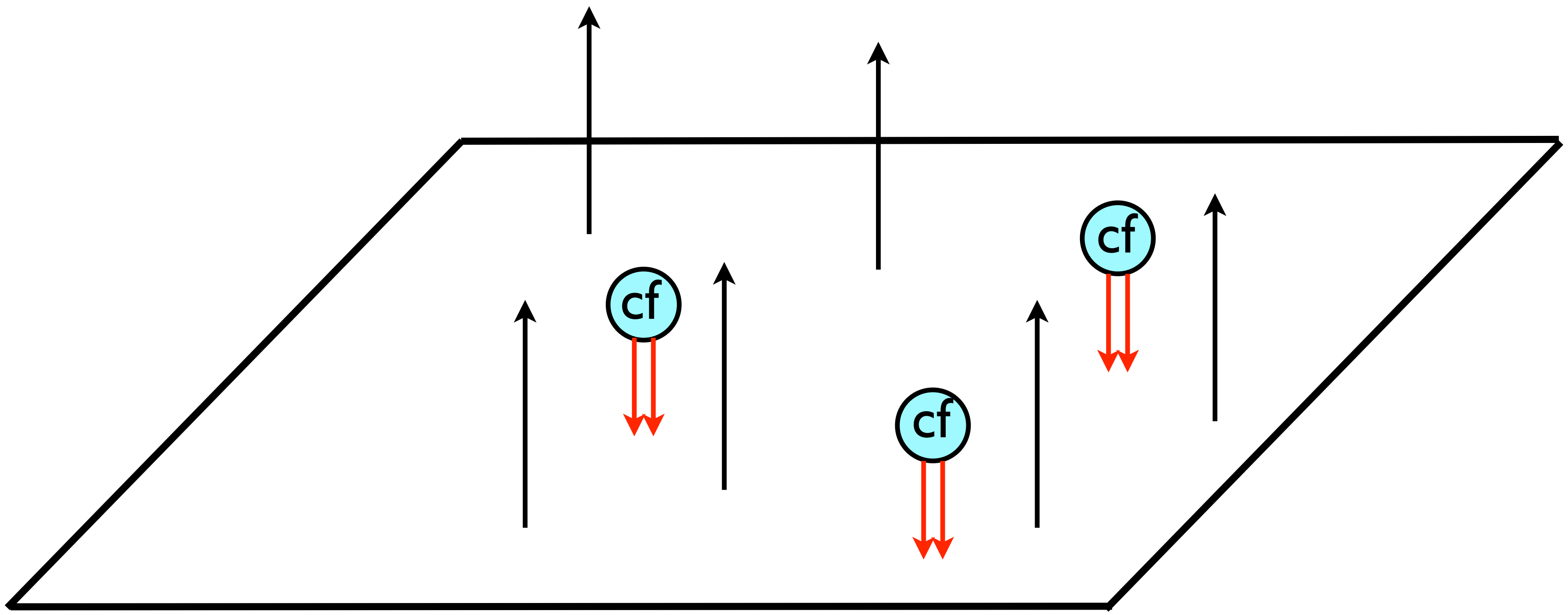


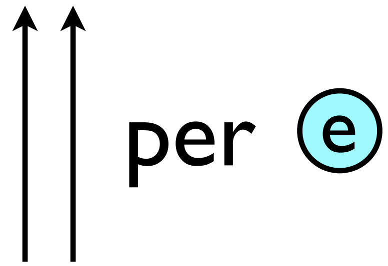
per e

Zero B field for cf

ν=1/2 state

Halperin Lee Read





Zero B field for cf

CFs form a gapless Fermi liquid: HLR field theory

HLR field theory

$$\mathcal{L} = i\psi^\dagger(\partial_0 - iA_0 + ia_0)\psi - \frac{1}{2m}|(\partial_i - iA_i + ia_i)\psi|^2 + \frac{1}{2}\frac{1}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$

$$b = \nabla \times a = 2 \times 2\pi\psi^\dagger\psi$$

Can be understood as a low-energy effective theory.

Two main features:

- number of CFs = number of electrons
- Chern-Simons action for gauge field *a*

Both comes with the flux attachment procedure

Halperin, Lee, Read 1993

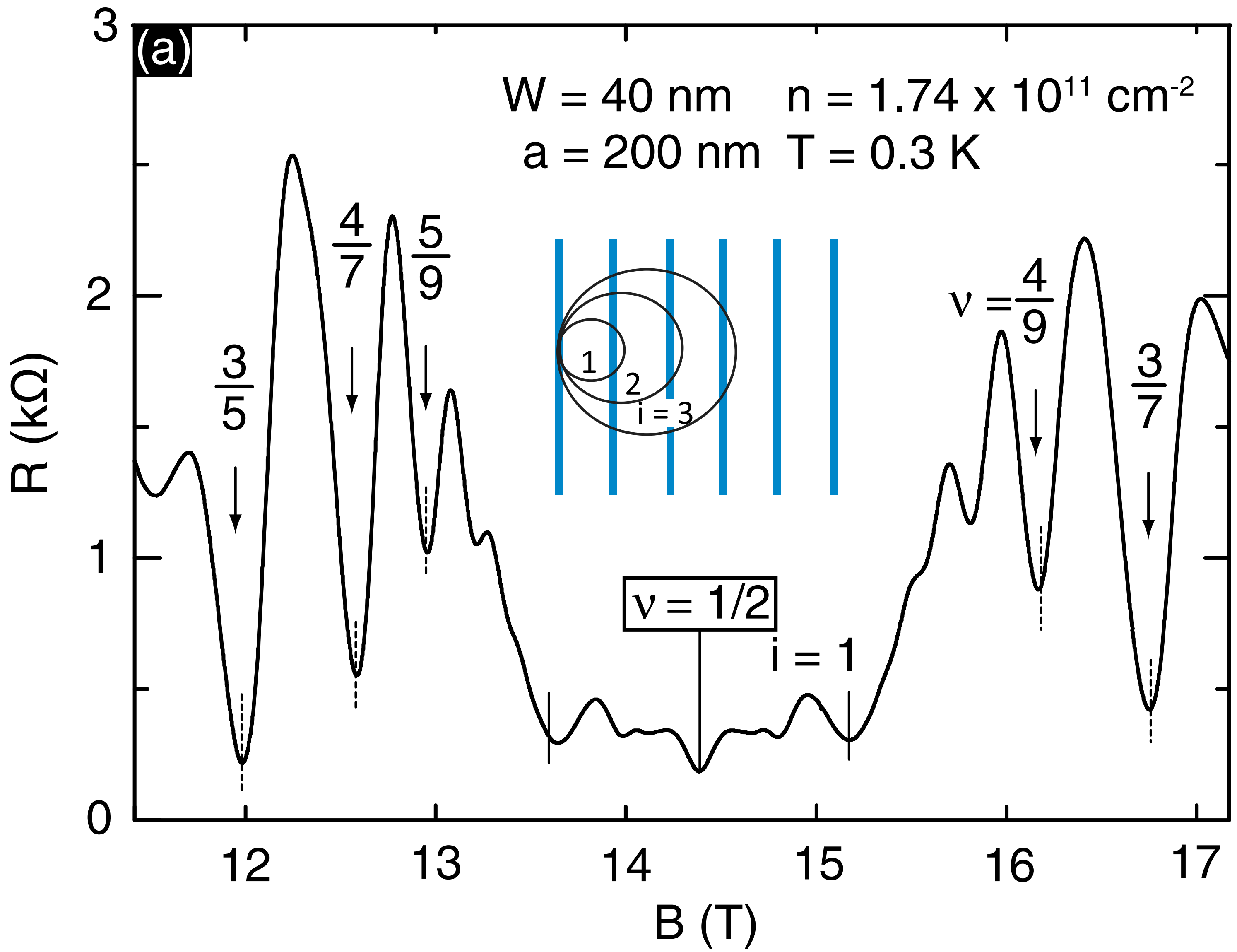
End of flux attachment

Is the composite fermion real?

- The most amazing fact about the composite fermion is that it exists!
- One way to detect the composite fermion:
  - go slightly away from half filling: composite fermions live in small magnetic field and move in large circular orbits

Kang et al, 1993

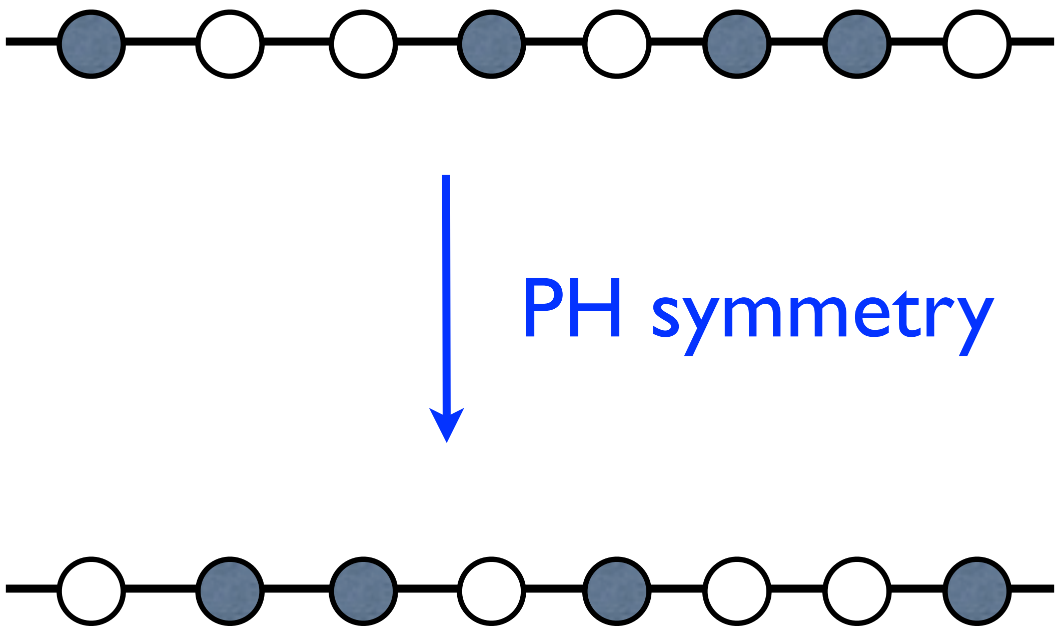
How real are the CFs ?



(Shayegan's group, 2014)

- Despite its success, the HLR theory suffers from a flaw: lack of particle-hole symmetry

Particle-hole symmetry



$$\Theta|\text{empty}\rangle = |\text{full}\rangle$$

$$\Theta c_k^\dagger \Theta^{-1} = c_k$$

$$\Theta\, i\, \Theta^{-1} = -i$$

$$\nu \to 1 - \nu \qquad \nu = \frac{1}{2} \to \nu = \frac{1}{2}$$

In the LLL limit: exact symmetry the Hamiltonian

Girvin 1984

PH symmetry in the CF theory

PH conjugate pairs of FQH states

$$\nu = \frac{n}{2n+1}$$

$$\nu = \frac{n+1}{2n+1}$$

ν=1/3

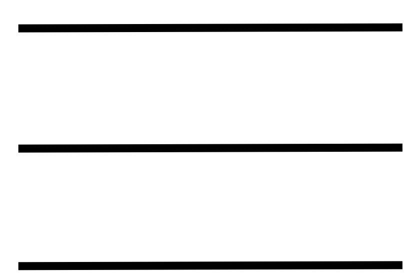
ν=2/3

PH symmetry in the CF theory

PH conjugate pairs of FQH states

$$\nu = \frac{n}{2n+1} \qquad\qquad \nu = \frac{n+1}{2n+1}$$

ν=2/5



ν=3/5

PH symmetry in the CF theory

PH conjugate pairs of FQH states

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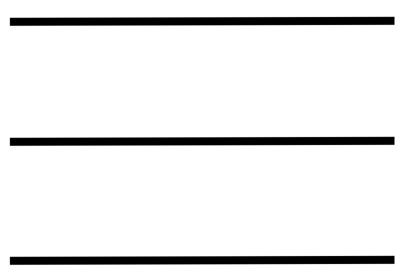

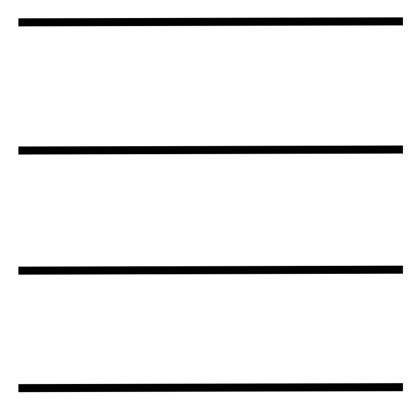

ν=3/7 ν=4/7

PH symmetry in the CF theory

PH conjugate pairs of FQH states

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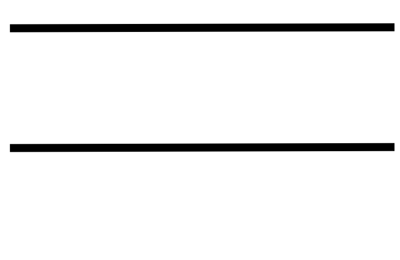
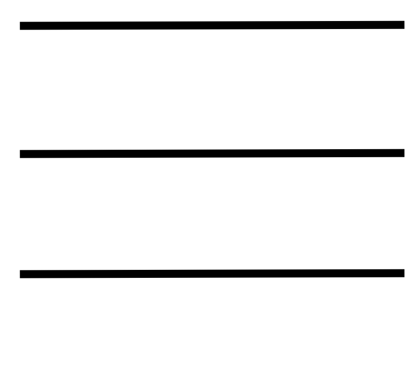


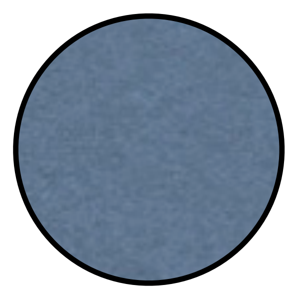
ν=3/7



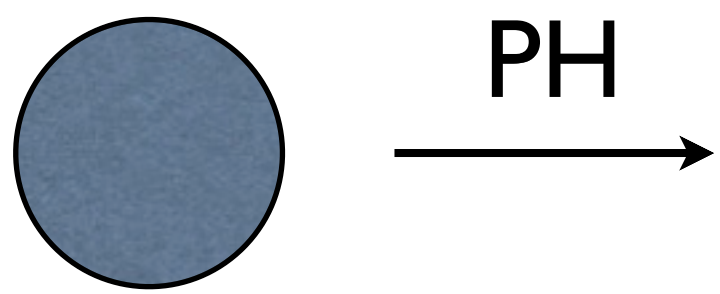
ν=4/7

CF picture does not respect PH symmetry

PH symmetry of CF Fermi liquid?



PH symmetry of CF Fermi liquid?



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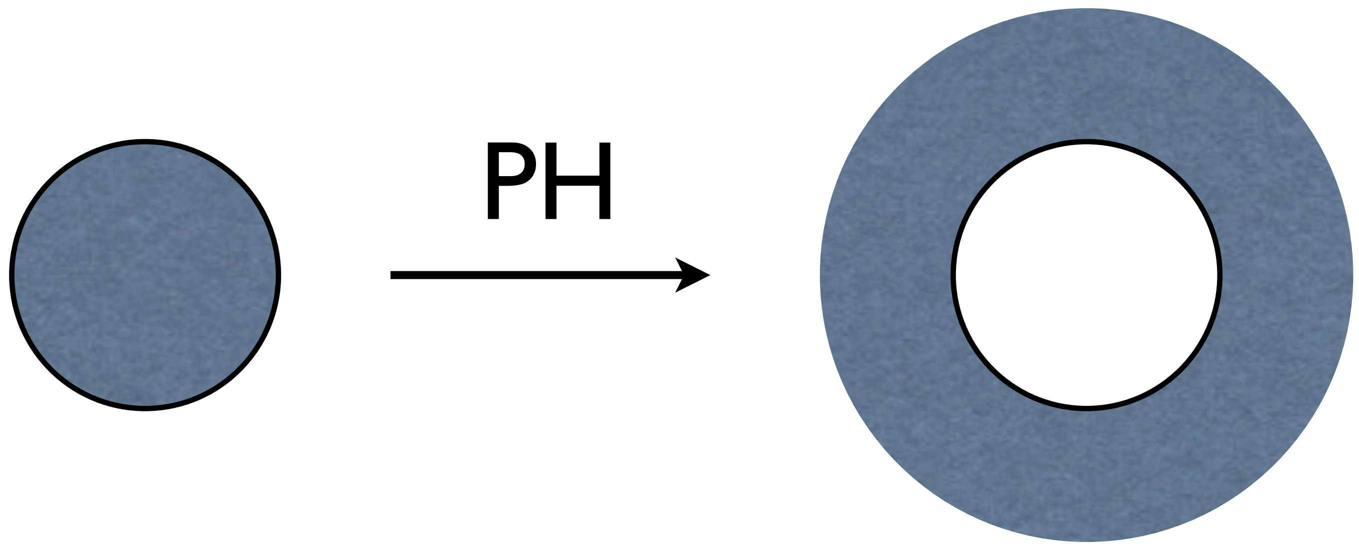

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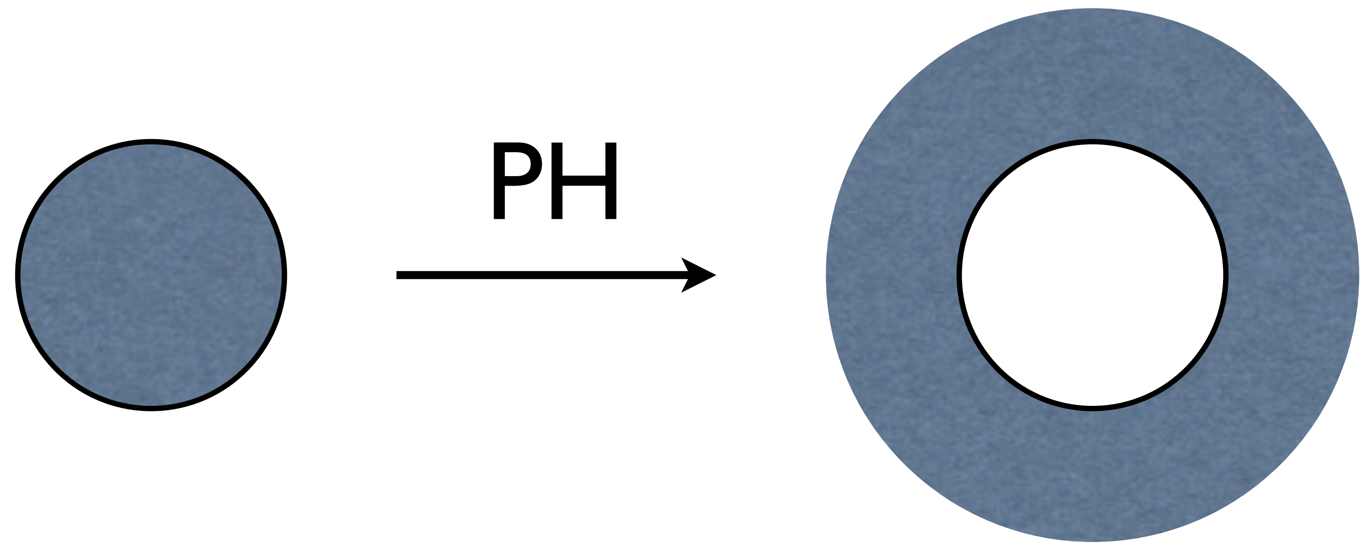

Related: no simple description of the anti-Pfaffian

- The particle-hole asymmetry of the HLR theory has been noticed early on 1997-1998 Kivelson, D-H Lee, Krotov, Gan
- The issue periodically came up, but no conclusive resolution has emerged
    - 2007: anti-Pfaffian state Levin, Halperin, Rosenow; Lee, Ryu, Nayak, Fisher

Why PH symmetry is difficult

- Particle-hole symmetry is not a symmetry of nonrelativistic fermions in a magnetic field (TOE)
- Only appears after projection to the lowest Landau level (SM)
- Not realized as a local operation on the fields

Three possibilities

Three possibilities

- Hidden symmetry of HLR theory

Three possibilities

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    - careful analysis of Kivelson et al (1997) showed no sign of it

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- Spontaneous breaking of particle-hole symmetry
  Barkeshli, Fisher, Mulligan 2015.

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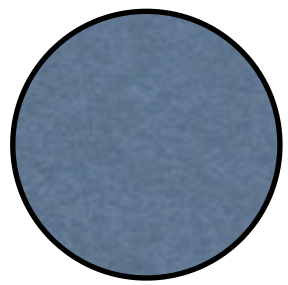
  - By now: considerable numerical evidence against it

Three possibilities

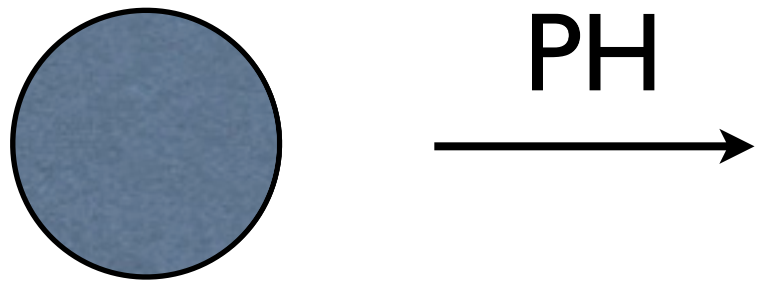
- Hidden symmetry of HLR theory
  - careful analysis of Kivelson et al (1997) showed no sign of it
- Spontaneous breaking of particle-hole symmetry
  Barkeshli, Fisher, Mulligan 2015.
  - By now: considerable numerical evidence against it
- The correct theory is a different theory, which realizes particle-hole symmetry in an explicit way

Dirac composite fermion



DTS, arXiv:1502.03446

Dirac composite fermion



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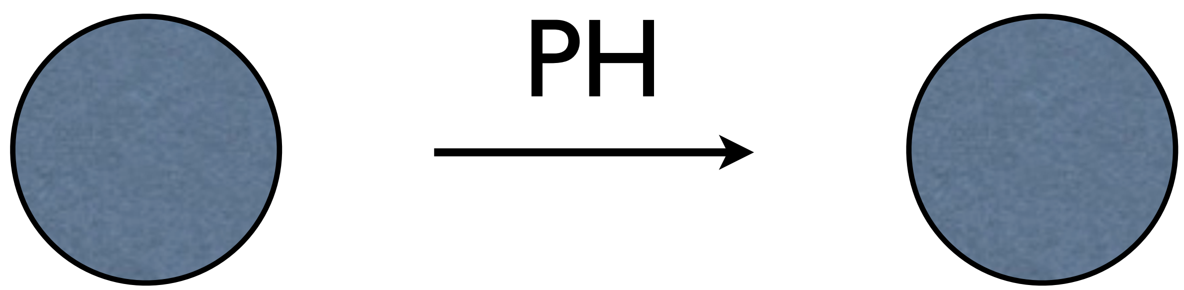
Dirac composite fermion



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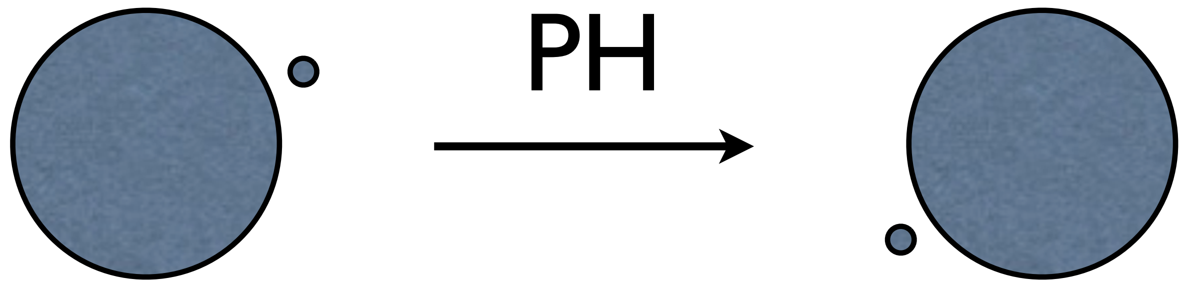

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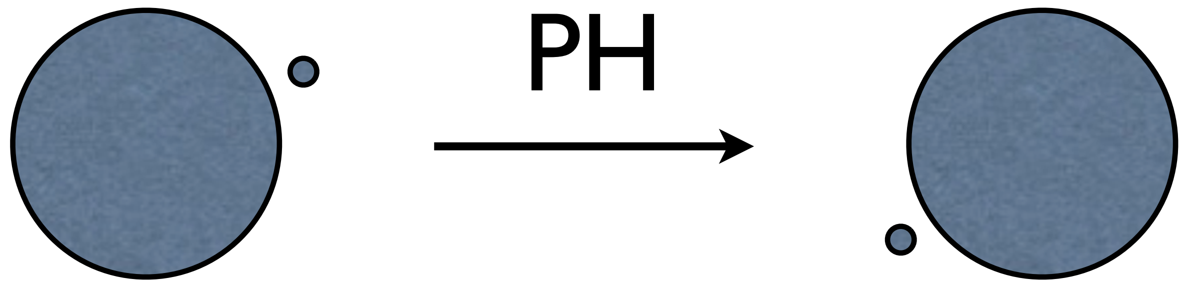

The composite fermion is a massless Dirac fermion
Particle-hole symmetry acts as time reversal

$$\mathbf{k} \to -\mathbf{k}$$

$$\psi \to i\sigma_2\psi$$

DTS, arXiv:1502.03446

First hint of Dirac nature of CFs

$$\nu = \frac{n}{2n+1} \longrightarrow \nu_{\mathrm{CF}} = n$$

$$\nu = \frac{n+1}{2n+1} \longrightarrow \nu_{\mathrm{CF}} = n+1$$

First hint of Dirac nature of CFs

$$\nu = \frac{n}{2n+1}$$

$$\nu = \frac{n+1}{2n+1}$$

$$\nu_{\mathrm{CF}} = n + \frac{1}{2} \; ?$$

First hint of Dirac nature of CFs

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$$\nu_{\mathrm{CF}} = n + \frac{1}{2} \; ?$$

$$\nu = \frac{n+1}{2n+1}$$

CFs form an IQH state at half-integer filling factor: must be a massless Dirac fermion

IQHE in graphene

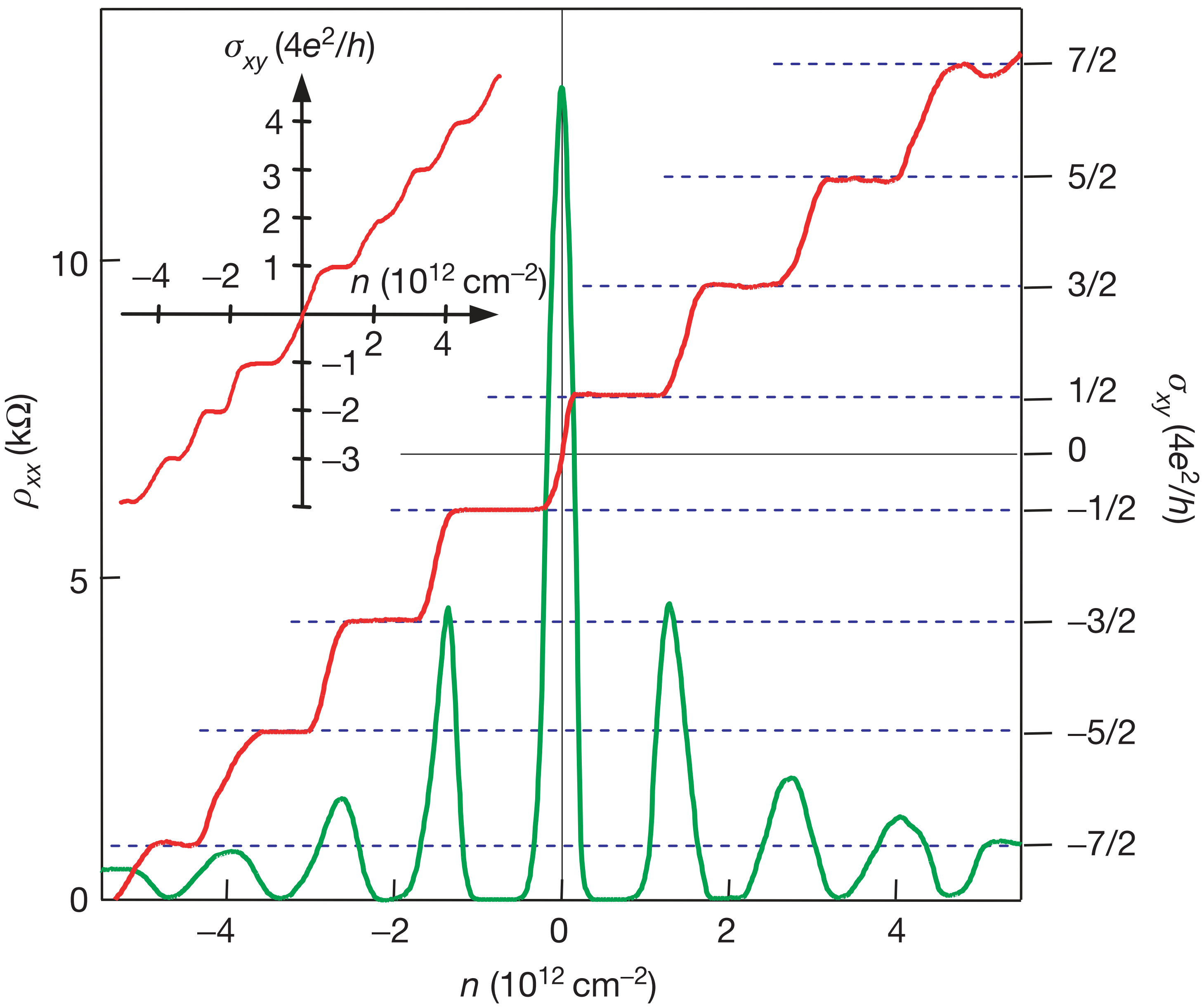
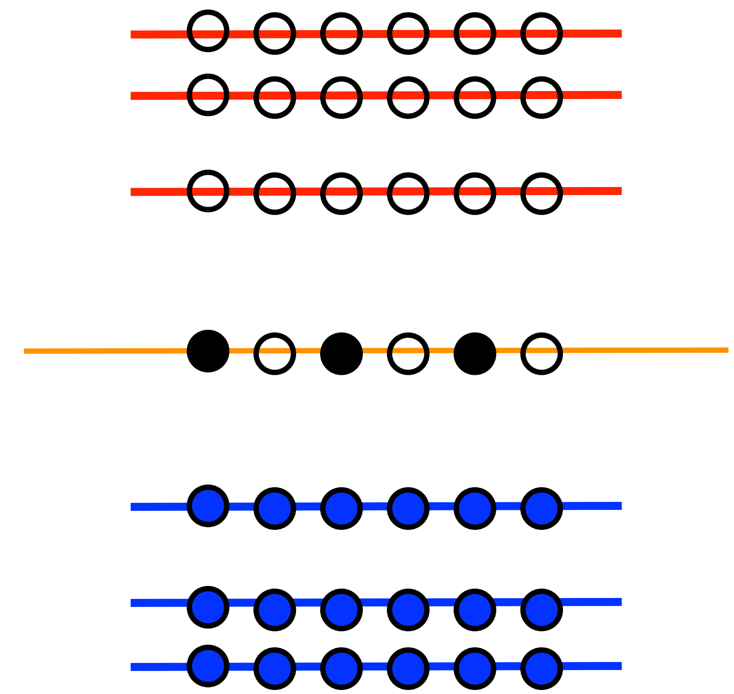


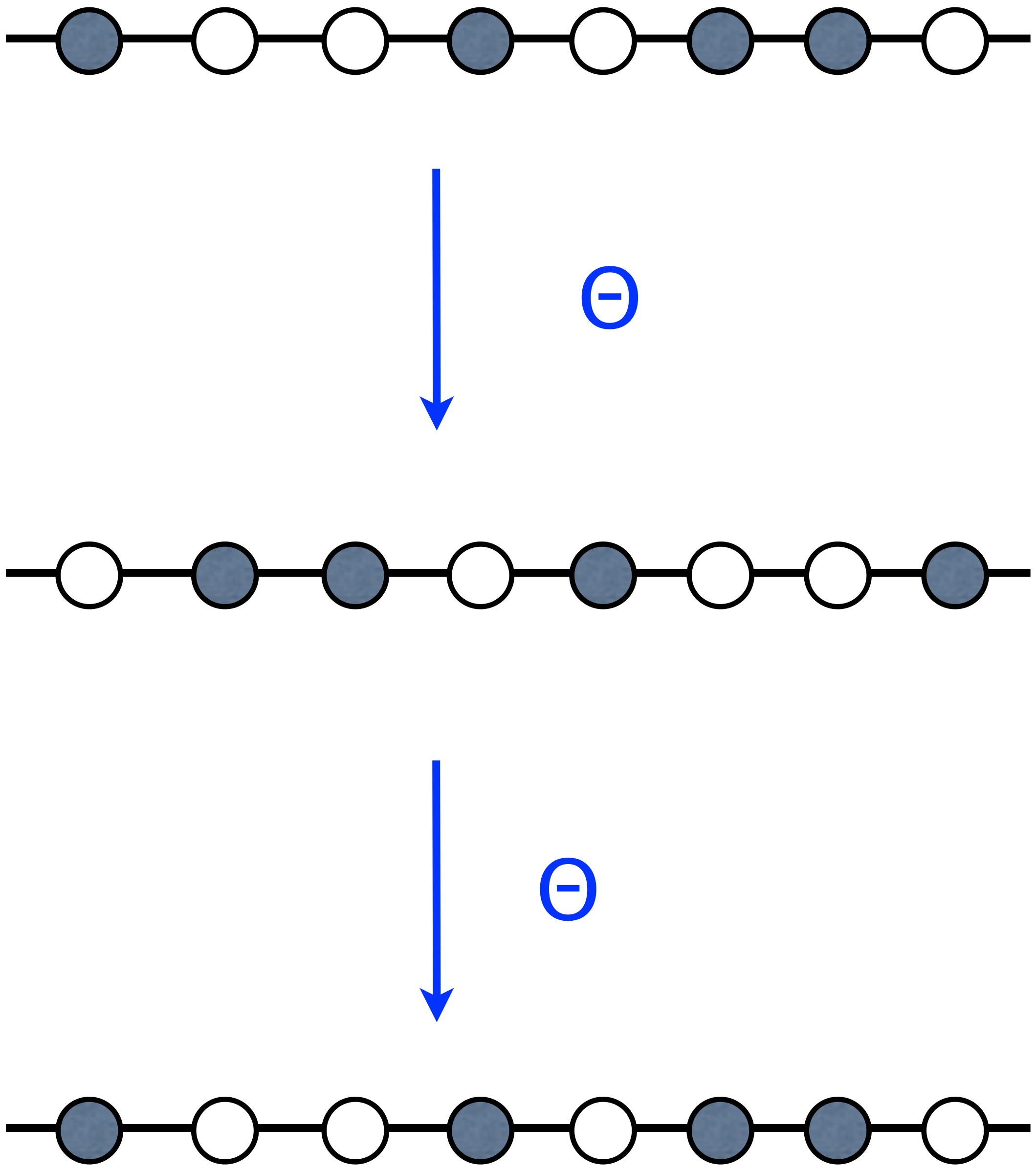
Figure 4 | QHE for massless Dirac fermions. Hall conductivity σ_{xy} and longitudinal resistivity ρ_{xx} of graphene as a function of their concentration at $B = 14$ T and $T = 4$ K. $\sigma_{xy} \equiv (4e^2/h)\nu$ is calculated from the measured

$$\sigma_{xy} = \left(n + \frac{1}{2}\right)\frac{e^2}{2\pi\hbar}$$



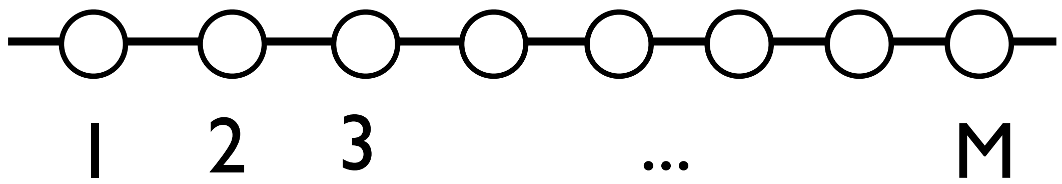
Novoselov et al 2005

(Particle-hole)^2



$$\Theta^2 = \pm 1$$

Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich 1508.04140
Levin, Son 2015 (unpublished)



$$\Theta|\mathrm{empty}\rangle = |\mathrm{full}\rangle = c_1^\dagger c_2^\dagger \cdots c_M^\dagger |\mathrm{empty}\rangle$$

$$\Theta c_k^\dagger \Theta^{-1} = c_k$$

anti-unitary

Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich 1508.04140
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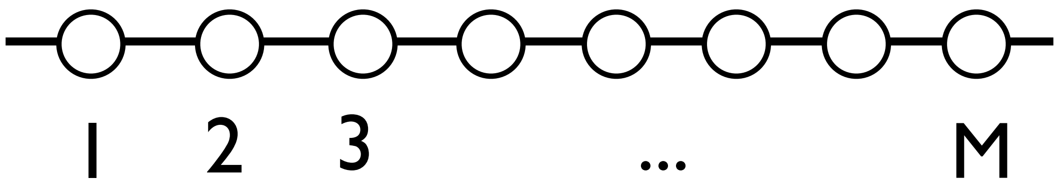

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anti-unitary

$$\Theta^2 = (-1)^{M(M-1)/2}$$

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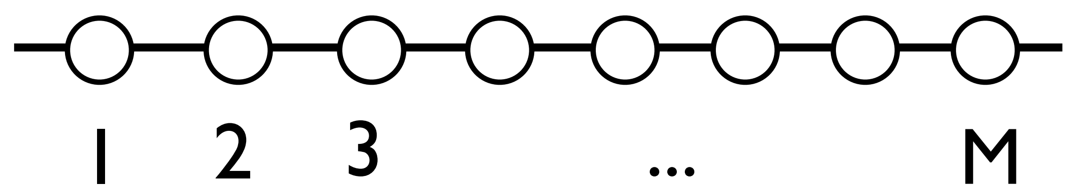

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anti-unitary

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$$M = 2N_{\mathrm{CF}}$$

$$\boxed{\Theta^2 = (-1)^{N_{\mathrm{CF}}}}$$

Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich 1508.04140
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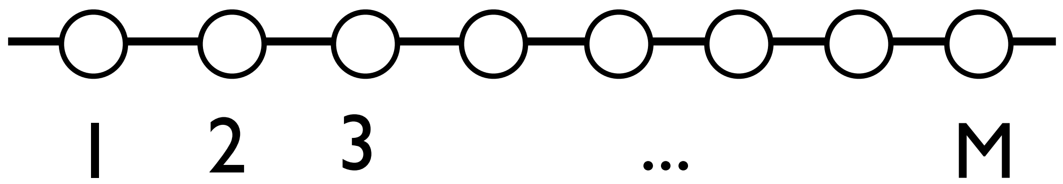


$$\Theta|\text{empty}\rangle = |\text{full}\rangle = c_1^\dagger c_2^\dagger \cdots c_M^\dagger |\text{empty}\rangle \qquad \Theta c_k^\dagger \Theta^{-1} = c_k$$

anti-unitary

$$\Theta^2 = (-1)^{M(M-1)/2}$$

$$M = 2N_{\mathrm{CF}} \qquad \boxed{\Theta^2 = (-1)^{N_{\mathrm{CF}}}}$$

Θ acts as time reversal on the Dirac composite fermion

$$\psi \to (i\sigma_2)\psi \to (i\sigma_2)^2\psi = -\psi$$

Problem with flux attachment

- For an even number of orbitals on the LLL,

$$\Theta^2|\mathrm{any}\rangle = (-1)^{M/2}|\mathrm{any}\rangle$$

- independent of the number of electrons in |any>
- Number of CFs = half the degeneracy of the LLL
  - does not coincide with the number of electrons away from half filling
- This goes against the philosophy of flux attachment

Tentative picture

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu + 2ia_\mu)\psi + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda + \frac{1}{8\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

- Instead of flux attachment, the CF appears through particle-vortex duality
- A known duality in the case of bosons, but in the case of fermion would be new
- ψ is a massless Dirac fermion
  - *ada* and fermion mass term would break particle-hole symmetry
- Jain sequences not affected

Particle-vortex duality

original fermion	composite fermion
magnetic field	density
density	magnetic field

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu + 2ia_\mu)\psi + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda + \frac{1}{8\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

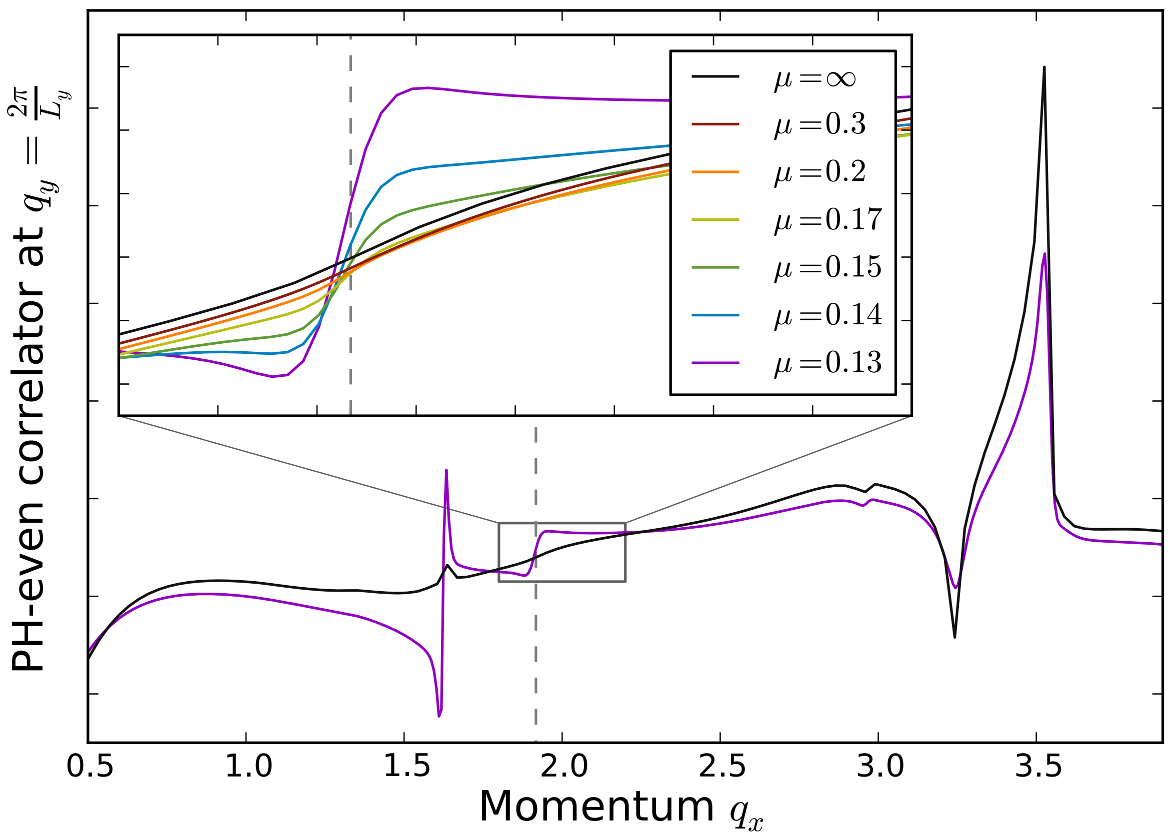
$$j^\mu = \frac{\delta S}{\delta A_\mu} = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda + \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$$

$$\frac{\delta S}{\delta a_0} = 0 \longrightarrow \langle \psi\bar{\gamma}^0\psi\rangle = \frac{B}{4\pi}$$

Consequences of Dirac CF

If $\hat{O}$ is particle-hole symmetric
i.e., $\hat{O} = (\rho - \rho_0)\nabla^2\rho$

$$\langle -\mathbf{k}|\hat{O}|\mathbf{k}\rangle = 0$$



Suppression of Friedel oscillations

Geraedts et al, 2015

Consequences of PH symmetry

$$\mathbf{j} = \sigma_{xx}\mathbf{E} + \sigma_{xy}\mathbf{E} \times \hat{\mathbf{z}} + \alpha_{xx}\boldsymbol{\nabla}T + \alpha_{xy}\boldsymbol{\nabla}T \times \hat{\mathbf{z}}$$

conductivities

thermoelectric coefficients

- At exact half filling, in the presence of particle-hole symmetric disorders

$$\sigma_{xy} = \frac{e^2}{2h} \qquad \alpha_{xx} = 0$$

HLR $\rho_{xy} = \dfrac{2h}{e^2}$

Potter, Serbyn, Vishwanath 2015

Taketani’s three stages

- Phenomenon: fractional quantum Hall plateaux
- Substance: composite fermion
- Essence: fermionic particle-vortex duality ?

How to get to the Essence

- Lowest Landau Level algebra from Dirac fermions Shankar Murthy
- Duality in surfaces of topological insulators at zero magnetic field: free fermion=QED_3? Metlitskii, Vishwanath; Wang, Senthil; Alicea, Essin, Mross, Motrunich...
- From supersymmetric duality Kachru et al
- General principles: anomaly matching, spin/charge relation Seiberg, Witten
- Experimental progress Wei Pan, W. Kang...